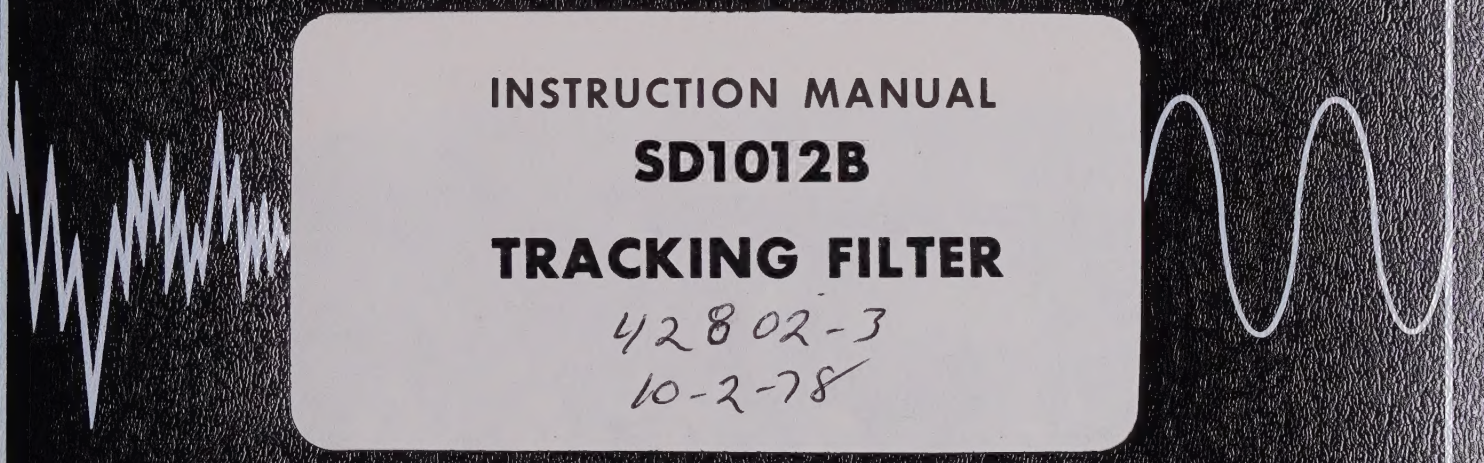




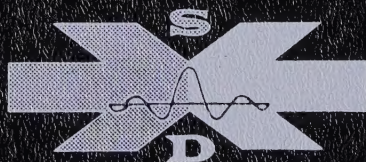
INSTRUCTION MANUAL

SD1012B

TRACKING FILTER

42802-3

10-2-78



SPECTRAL DYNAMICS CORPORATION

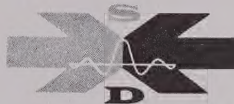


INSTRUCTION MANUAL SD1012B TRACKING FILTER

42802-3
10-2-78

U. S. PATENT NO. 3,018,439

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October 1977



MODEL SD1012B TRACKING FILTER

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DWG. NO. 11386	Schematic/Component Layout (4 Sheets)	Rear

APPENDIXES

Appendix A:

Calculating Values of Sweep Rate Control Resistors Used in PSD Plug-In

Assemblies (11312)	Rear
------------------------------	------

PS1:

"Some Analog Methods of Power Spectral Density Analysis"

by Laurie R. Burrow Jr.	Rear
---------------------------------	------

PS2:

"Considerations In The Analysis of Arbitrary Waveforms"

by Anton C. Keller	Rear
------------------------------	------

PREFACE

RECEIVING INSPECTION PROCEDURE

The Model SD1012B Tracking Filter is shipped fully assembled, packed in a polystyrene case designed to protect it against all normal shipping hazards. Immediately upon receipt, inspect the exterior of the shipping case and note any visible damage. Keep all forms and invoices attached to the shipping case. Remove the instrument from the shipping case and test its functional operation in accordance with Section II of this manual. If shipping damage is apparent, file a claim with the carrier's claims agent and send a copy to Spectral Dynamics Corporation. Be sure to include the instrument name, model number, and serial number on all correspondence. Spectral Dynamics Corporation will advise the buyer what should be done; arrangements for repair or replacement will be made as applicable.

HARDWARE WARRANTY POLICY

All products manufactured by Spectral Dynamics are warranted against defects in materials and workmanship for one year from the date of delivery or, in the case of certain components listed in some of the operating manuals, for the specified period. We will repair or replace products which prove defective during the warranty period, providing the failure or damage has not been caused by misuses or abnormal operating conditions.

If the malfunction or a portion thereof is determined by Spectral Dynamics Corporation to have been caused by misuses or abnormal conditions of operation, an estimate of cost to repair will be submitted to the purchaser for approval before beginning any repair work.

Computers, computer accessories and other equipment made by other manufacturers and sold by Spectral Dynamics for use with its instruments are specifically excluded from this warranty. Spectral Dynamics encourages buyers of systems which include computer equipment to obtain Service Agreements with the computer manufacturer. We are happy to assist in this regard.

Liability under this warranty is limited to servicing and adjusting the instrument returned to the Spectral Dynamics Corporation factory, with transportation charges prepaid by the purchaser. NO OTHER WARRANTY IS EXPRESSED OR IMPLIED. WE ARE NOT LIABLE FOR CONSEQUENTIAL DAMAGES.

If a malfunction develops, notify the Customer Service Department of Spectral Dynamics Corporation or its representative in your area, giving details of the problem and the name, model and serial number of the equipment. Upon receipt of this information, service data or shipping instructions will be provided.

RETURN SHIPMENT PROCEDURE

On receipt of shipping instructions, forward the instrument prepaid via air freight to the factory or authorized repair station indicated on the instructions. Replace the instrument in its original shipping case, or surround with two to three inches of shock absorbing material and then pack it in a heavy-weight cardboard box, wooden box, or other strong container.

HARDWARE WARRANTY POLICY

All products manufactured by Spectral Dynamics are warranted against defects in materials and workmanship for one year from the date of delivery or, in the case of certain components listed in some of the operating manuals, for the specified period. We will repair or replace products which prove defective during the warranty period, providing the failure or damage has not been caused by misuse or abnormal operating conditions.

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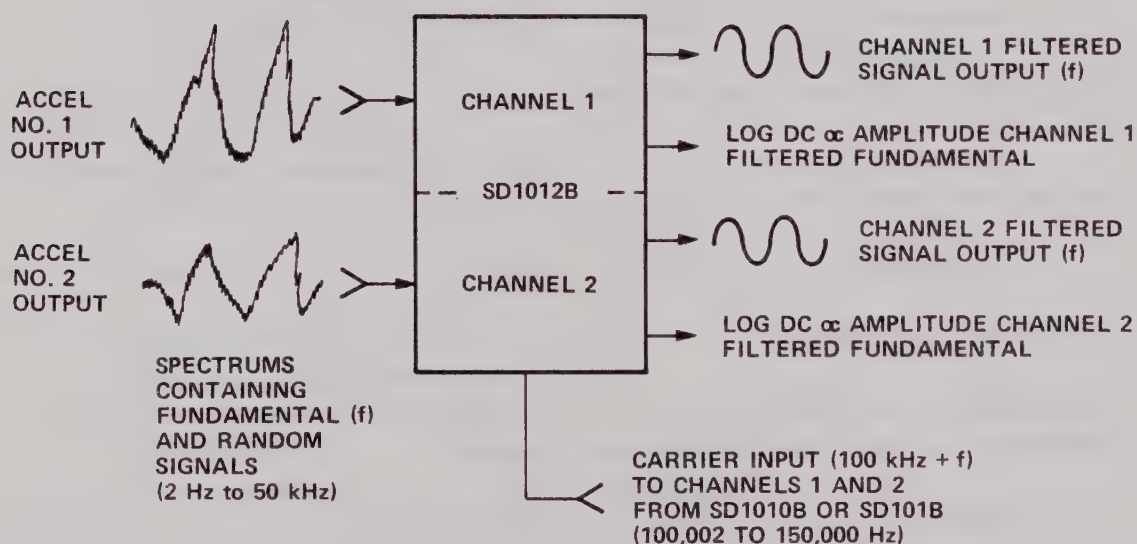
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SECTION I

GENERAL INFORMATION

1.1 INTRODUCTION

The Model SD1012B Tracking Filter is a solid-state, dual-channel, frequency-tuned bandpass filter. From a complex spectrum such as the output of an accelerometer, either or both channels of the SD1012B can select and filter out a single frequency. For example, the following illustration shows the Model SD1012B filtering the outputs from two accelerometers attached to a specimen undergoing vibration analysis.



Each channel also provides various a-c and d-c outputs for phase measurement, recording, and other applications. A separate front-panel meter for each channel continuously displays the amplitude of the tuned signal in both volts rms and relative dB level.

An optional sine reject capability is available in either Channel 1 (Model SD1012B-1) or both Channels 1 and 2 (Model SD1012B-2). Sine reject operation is used during combined sine/random testing. (Refer to paragraph 2.2.4 in Section II.)

1.2 SPECIFICATIONS

1.2.1 INPUTS PER CHANNEL

SIGNAL

Frequency Range 2 Hz to 50 kHz

Sensitivity 31.6 mV to 10 Vrms full scale in 10 dB steps

Impedance 100 k Ω shunted by 200 pF (nominal)

CARRIER

Amplitude 2.4V p-p (nominal)

Impedance 80 k Ω

Frequency 100,002 Hz to 150 kHz

DC FREQUENCY (Auto Filter Switching)

Amplitude 0 to 1V maximum over frequency range of operation

Impedance 50 k Ω

1.2.2 OUTPUTS PER CHANNEL

FILTERED SIGNAL (Unity Gain)

(Referenced to 1 Vrms input RANGE switch position)

Frequency 2 Hz to 50 kHz

Frequency Response ± 0.3 dB from 5 Hz to 30 kHz
 ± 0.6 dB from 2 Hz to 50 kHz
From 0 dB to -60 dB of full scale

Amplitude Unity gain ± 0.2 dB into open circuit, 10 Vrms maximum

Impedance Less than 1 k Ω

Dynamic Range 76 dB (Overload to Noise Floor)

Linearity $\pm 5\%$ of reading $\pm 0.01\%$ of full scale, 0 dB to -60 dB at 400 Hz

FILTERED SIGNAL (10V Full Scale)

- Amplitude 10 Vrms $\pm$ 0.2 dB full scale for each range into open circuit
- Impedance 1 k Ω , 2 mA p-p maximum

LINEAR DC
(Referenced to 1 Vrms input RANGE switch position)

- Amplitude 10V $\pm$ 0.2V full scale, into open circuit
- Impedance 250 Ω , 3 mA maximum
- Linearity $\pm$ 5% of reading, $\pm$ 0.01% of full scale over 50 dB range at 400 Hz
- Frequency Response $\pm$ 0.3 dB from 5 Hz to 30 kHz
 $\pm$ 0.5 dB from 2 Hz to 50 kHz
From 0 to -50 dB of full scale

100 kHz FILTERED SIGNAL

- Amplitude 1.8 Vrms $\pm$ 0.5 dB full scale into open circuit
- Impedance <50 Ω , 3 mA maximum
- Frequency Range, Frequency Response, Dynamic Range, and Linearity same as Filtered Signal (Unity Gain) Referenced to 1 Vrms input RANGE switch position

LOG DC

- Amplitude Full scale to -50 dB into open circuit, referenced to 1 Vrms input RANGE switch position
- Sine Mode 0.2V to -0.1V (nominal)
(100 mV/dec.)
- PSD Mode 0.4V to -0.2V (nominal)
(200 mV/dec.)
- Impedance 500 Ω
- Linearity Same as LIN DC $\pm$ 0.25 dB
- DC Attenuator Range 0 dB to -40 dB from full scale in 10 dB steps $\pm$ 0.25 dB

SINE REJECT

(Optional in Models SD1012B-1 and SD1012B-2)

Amplitude	Unity gain ± 0.2 dB, 3.16 Vrms maximum into open circuit
Accuracy	± 0.25 dB
Dynamic Range	30 dB minimum referenced to 1 Vrms input RANGE switch position
Impedance	(10 Hz to 5 kHz) Less than 1 k Ω

METER

Frequency Range, Frequency	Same as FSO (Unity Gain)
Response Linearity	$\pm 2\%$ of full scale and Multiplier Position referenced to 1 Vrms input RANGE switch position

1.2.3 MISCELLANEOUS

SELECTIVITY	2 plug-in crystal filters (per channel)
Bandwidths	Available: 1.5, 2, 5, 10, 20, 50, 100 & 200 Hz (others on special order)
Shape Factor	Nominally 4 to 1 (60 dB/3 dB BW)
CALIBRATION	Internal calibration reference
PSD PARAMETERS	Plug-in networks for each filter program averaging time & sweep rate (using SD104A or SD114 Sweep OSC) for 4 PSD confidence levels; bandwidth normalization set by front panel control. Average time constant accuracy $\pm 10\%$.
PHASE ADJUSTMENT	Front panel control for each plug-in filter provides 0° to 120° phase adjust; internal 0°/180° switch also provided for each filter.
SINE AVERAGING	4 front panel switch-selectable averaging times of 0.01, 0.1, 0.5 and 1 second $\pm 10\%$
INPUT/OUTPUTS	BNC Connectors

POWER 105–125/210–250 Vac 50-60 Hz, at $\approx 190\text{W}$;
input line voltage switch-selectable at rear of
chassis

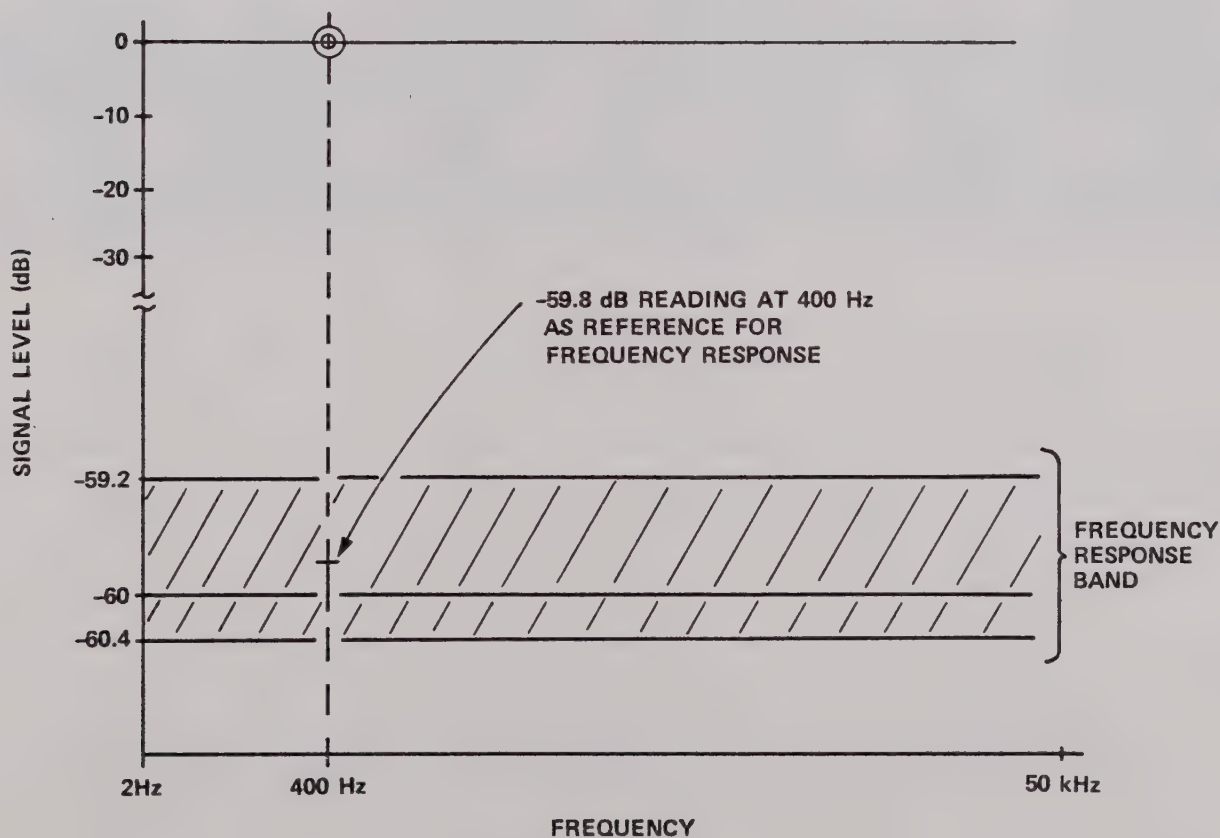
DIMENSIONS

Height $5\frac{1}{4}"$ (13,3 cm)
Width 19" (48,3 cm)
Depth 17" (43,2 cm)
Shipping Weight 22 lbs (9,97 kg)

1.2.4 INTERPRETATION OF LINEARITY AND FREQUENCY RESPONSE SPECIFICATIONS

The following graph illustrates the cumulative linearity and frequency response errors possible over the frequency range of the instrument. As specified in paragraph 1.2.2, the FSO (filtered signal output) linearity is "5% of reading $\pm 0.01\%$ of full scale, 0 dB to -60 dB, at 400 Hz". Therefore, the total linearity error possible at full scale is ± 0.425 dB, computed as follows: ± 0.424 dB (5% of reading) + $\approx \pm 0.001$ dB ($\pm 0.01\%$ of full scale).

At a signal level of -60 dB from full scale, if the 400-Hz output reads +0.2 dB above true -60 dB, then the SD101B can read ± 0.6 dB from this reference for any frequency from 2 Hz to 50 kHz.



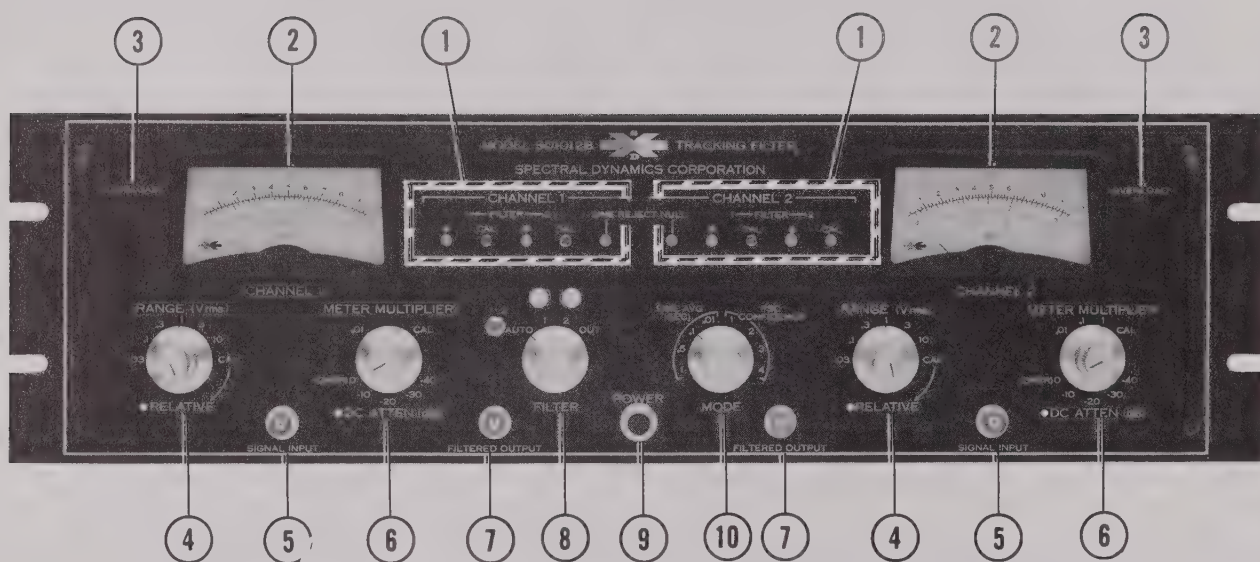


Figure 1-1. Front Panel

1.3 FRONT PANEL FUNCTIONAL DESCRIPTION

The descriptions given in the following paragraphs are keyed by circled numbers to their associated front-panel control, meter, indicator, or connector as shown in Figure 1-1, Front Panel. All interacting controls, indicators, etc., have been grouped to maintain continuity of description.

① Filter Calibration Group

FILTER CAL

Used during system calibration to normalize the gain of the crystal filters over the two selectable bandwidths.

FILTER ϕ

Used during system calibration to normalize the output phase of the signal through each filter relative to the other, or the output phase relative to that of another unit (SD101B Dynamic Analyzer or another SD1012B).

SINE REJECT NULL

Used during combined sine/random testing to cancel the sine component from the data input, leaving the random component which is available at SINE REJECT output connector on rear chassis. (Operational only on Models SD1012B-1, -2.)

② Meter, Signal Amplitude

Indicates the rms voltage amplitude and relative dB level of the input signal at the TUNING INPUT frequency in ranges determined by the RANGE (Vrms) and METER MULTIPLIER switch settings. The top 0—10 scale is used for RANGE settings of .1, 1, and 10; the center 0—3 scale is used for RANGE settings of .03, .3, and 3. The bottom -20 dB to 0 dB scale indicates the level of the measured voltage in dB referenced to full scale as determined by the RANGE (Vrms) and METER MULTIPLIER switch settings. CAL mark at extreme right-hand side is read as 10 on top scale; used during system calibration.

③ OVERLOAD Indicator

Lights if input signal exceeds selected RANGE (Vrms) by 12 dB. If lighted, operator should select next higher range until OVERLOAD indicator extinguishes.

4 RANGE (Vrms)/RELATIVE

This dual control performs the following functions:

RANGE (Vrms)

Outside (larger) knob selects the SIGNAL INPUT voltage range in volts rms of the instrument, except in CAL position; when set to CAL, a square wave (power line frequency) calibrated to provide full-scale deflection (CAL or 10) is applied as the input signal. If the SIGNAL INPUT amplitude exceeds the selected range by 12 dB, the OVERLOAD indicator lights. However, INPUT overload protection is provided on all input ranges.

RELATIVE

Inside (smaller) knob turns a potentiometer that provides 0 to -10 dB variable gain adjustment of the input amplifier. Can be used when making relative amplitude measurements or to adjust system gain during routine calibration. Switch detent at extreme clockwise position disconnects gain potentiometer from circuit for normal operation and during calibration.

5 SIGNAL INPUT

Used for applying data signals to be filtered to the instrument; connected in parallel with SIGNAL INPUT connector on rear chassis. When instrument is rack-mounted in a system, input signals are normally applied to the rear chassis connector and this connector can be used for signal monitoring.

6 METER MULTIPLIER/LOG DC ATTEN (dB)

This dual control performs the following functions:

METER MULTIPLIER

Outside (larger) knob setting determines the computing factor to be used with the Signal Amplitude Meter reading to obtain actual voltage magnitude of the SIGNAL INPUT voltage.

LOG DC ATTEN (dB)

Inside (smaller) knob provides 0 to -40 dB attenuation, in 10 dB increments, of the LOG DC output (rear chassis panel) for external recorder calibration. When not calibrating, set LOG DC ATTEN (dB) to (OPER) 0.

7 FILTERED OUTPUT

The FSO (filtered signal out) is phase and amplitude coherent with the input signal at the tuning frequency; the FSO bandwidth equals that of the selected crystal filter. This connector is in parallel with FIL SIG UNITY GAIN OUTPUT connector on the rear chassis panel.

8 FILTER Switch, Indicators, and 1-2 AUTO Crossover Adjust

FILTER Switch

Used to manually select any one of two internal crystal filter channels. When set to AUTO, the crystal filter channels can be selected automatically as a function of system tuning frequency by application of a d-c voltage proportional to frequency at rear chassis connector CH 1-2 DC $\propto$ FREQ INPUT. (See 1-2 AUTO Crossover Adjust below.) When set to OUT, the SIGNAL INPUT connectors (both front panel and rear chassis) are connected directly to the FILTERED SIG OUT (front panel) and UNITY GAIN (rear chassis) connectors, thus completely bypassing the instrument. However, the SIGNAL INPUT is still applied to the instrument circuit; the meter monitors signal level, and the 100-kHz FIL I.F., LIN DC and LOG DC outputs are available; filter selection is in automatic mode.

Filter Indicators

Filter indicator 1 or 2 lights to signal which crystal filter channel is being used; these indicators operate in both AUTO and manual filter select modes.

1-2 AUTO Crossover Adjust

Used in AUTO filter select mode to set frequency at which automatic switching shall occur between filter channels 1 and 2.

9 POWER

Push button type switch/indicator used to turn instrument on and off. Push button glows red when power is on.

10 MODE

Used to select the desired averaging time constants for filtering the linear and log d-c output voltages when analyzing sine or random (PSD) data signals. When analyzing periodic data, SINE AVG (SEC) times of .01, .1, .5, and 1 second are available; for random signals, any one of four PSD CONFIDENCE levels can be selected. (Refer to theory of operation of these functions in Section III.)

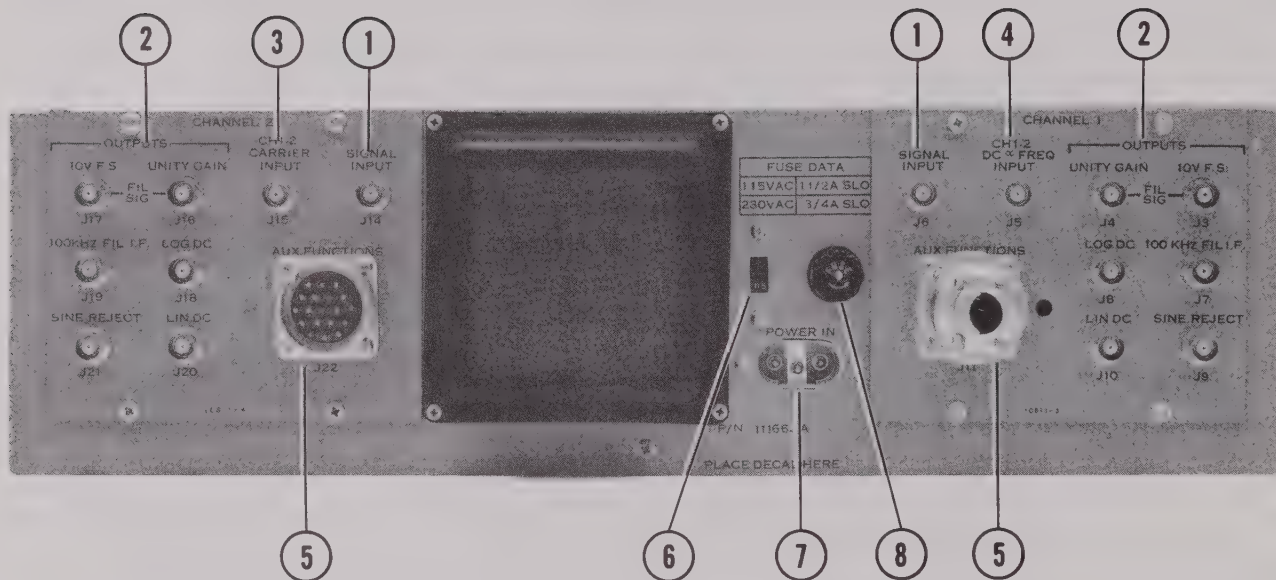


Figure 1-2. Rear Chassis

1.4 REAR CHASSIS FUNCTIONAL DESCRIPTION

The descriptions given in the following paragraphs are keyed by circled numbers to their associated rear-chassis switch or connector as shown in Figure 1-2, Rear Chassis.

Where nomenclature is common to both channels, the connector numbers in parenthesis are interpreted: (Channel 1, Channel 2).

① SIGNAL INPUT (J6, J14)

Used for applying data signals to be filtered to the instrument; connected in parallel with SIGNAL INPUT connector on front panel.

② OUTPUTS Group

FIL SIG OUTPUTS

Used for connecting the FSO to external recording equipment, to servo control equipment in a shaker system, and for closing the tracking servo loop when used in a mechanical impedance system. (Refer to application in Section II.)

UNITY GAIN (J4, J16)

These connectors are connected in parallel with associated channel front-panel FILTERED OUTPUT connector.

10V F.S. (J3, J17)

These outputs provide 10 volts rms full scale over the selected RANGE (Vrms); i.e., with full-scale input on any range, this output is 10 Vrms. Note that this output is present even with the FILTER switch set to OUT.

100-kHz FIL I.F. (J7, J19)

Provides a 100-kHz filtered signal that is amplitude and phase coherent with the data input signal at the tuning frequency; the bandwidth equals that of the selected crystal filter passband. This signal is used for phase comparison in mechanical impedance and mode study systems, for phase control, and cross-spectrum analysis.

LOG DC (J8, J18)

Provides a d-c voltage output proportional to the log of the amplitude of the data input signal at the tuning frequency, for connecting to an X-Y recorder.

SINE REJECT (J9, J21)

Used during combined sine/random testing to connect the output of the sine reject circuit to external control equipment. This signal contains only the random component of the input data spectrum, the sine component being rejected, over the bandwidth of the selected crystal filter passband. (Output available only on Models SD1012B-1, -2.)

LIN DC (J10, J20)

Provides a d-c voltage output that is linearly proportional to the amplitude of the data input signal at the tuning frequency.

3 CH 1-2 CARRIER INPUT (J15)

Accepts 2.4V p-p carrier signal (100,002 Hz to 150,000 Hz) supplied by a Model SD1010B Carrier Generator or SD101B Dynamic Analyzer.

4 CH 1-2 DC $\propto$ FREQ INPUT (J5)

Used for applying a d-c voltage proportional to the system tuning signal frequency, supplied by the sweep generator or a frequency log converter, for controlling the automatic filter-bandwidth and PSD confidence level switching circuitry. The input voltage applied must not exceed 1 volt over the frequency range of operation.

5 AUX FUNCTIONS (J11, J22)

Used for connecting interunit control functions when instrument is used in a mechanical impedance or PSD analysis system. See Tables 2-1 and 2-2 in Section II and the schematic diagram for system interconnection details. Refer also to the instruction manual for the system in which the instrument is installed.

6 115/230 Switch

Facilitates changeover of primary power input circuit for either 115V or 230V operation. Proper input voltage to be applied shows in window; screwdriver blade slot is up for 115V operation and down for 230V operation.

7 POWER IN (J19)

Accepts 3-wire plug and cable (supplied) for applying 115/230 Vac 50-60 Hz primary input power. Center terminal is chassis ground. Uses Belden No. 17258 cable.

8 FUSE

Contains 3AG type, SLO-BLO fuse; 1-½A for 115V operation or ¾A for 230V operation.

SECTION II

OPERATION

2.1 GENERAL

The Model SD1012B Tracking Filter (two-channel) was designed to be used as an integral part of a dual-channel Power Spectral Density (PSD) or Mechanical Impedance (MZ) system, or other multi-channel tracking filter system. Therefore, the operator should refer to the applicable system instruction manual, which includes installation and operation instructions for the Model SD1012B. See also Figures 1-1 and 1-2 in Section I of this manual for the description and location of all operating controls, indicators, and connectors.

The following preliminary setup, turn-on, and operating instructions are provided mainly for the user who has purchased the SD1012B as a separate instrument for integration into an existing system. In this case, it is assumed that a system interconnection diagram has been designed either by the user or by an SDC Engineering Representative, as applicable.

2.2 OPERATIONAL CHECKOUT PROCEDURE

The following procedure should be performed upon receipt of the instrument and after maintenance to ensure correct overall operation.

2.2.1 Preliminary Setup

- a. Check that the 115/230 switch on instrument rear chassis is set to proper input line voltage and that proper fuse is installed.
- b. Remove instrument top cover and filter section cover, and install crystal filter(s) in each channel.

NOTES

- 1) If only one crystal filter is used in each channel, plug it into FILTER 1 socket. If two filters are used, it is recommended that they be plugged into filter sockets 1 and 2, in order, from lower to higher bandwidth.

(Cont'd)

- 2) If PSD plug-in assemblies (11312-) are used, ensure that the 11312 assembly dash number corresponds to the associated filter channel bandwidth. For example: if a 5-Hz bandwidth filter is used in FILTER 1 socket, the Channel 1 PSD plug-in assembly must be marked 11312-5.

CAUTION

A PSD Plug-in Assembly (11312-) MUST BE installed in each Crystal Filter channel used or the instrument WILL NOT operate in PSD mode.

- c. Replace filter section cover and instrument top cover, and connect power cord (supplied) to primary power source.
- d. Ensure that plug P11 (SDC 11514) and P22 (SDC 11515) are inserted into AUX rear-chassis connectors J11 and J22. Figure 2-1 shows construction details for plugs P11 and P22.
- e. Connect the SD1010B CARRIER OUT, 2.4V P-P, to the SD1012B CH 1-2 CARRIER INPUT (J15).
- f. On SD1012B, press front-panel POWER switch/indicator; it should glow red indicating power is on. Allow at least 1 hour for crystal filters to warm up. Also apply power to SD1010B.

2.2.2 Filter Calibration

After the procedure outlined in paragraph 2.2.1 has been performed and the SD1012B has warmed up for at least 1 hour, use the following procedure to calibrate Channel 1 and then Channel 2.

CAUTIONS

- 1) The SD1012B (and SD1010B) internal CAL signal frequency is the power line frequency. Assuming 60 Hz line frequency, if a 150-Hz BW filter is calibrated using internal CAL signal, an error of $\approx 1\%$ would be introduced. Therefore, when calibrating a 100-Hz or higher BW filter, an external signal whose frequency $\geq \frac{1}{2}BW$

(Cont'd)

must be applied to the SD1012B SIGNAL INPUT and the SD1010B TUNING INPUT. In this case, set the SD1012B RANGE switch as required by the level of the external calibration signal, and adjust the SD1010B TUNING AMPL adjust for center-scale reading on SD1010B TUNING AMPLITUDE meter. Observe the following CAUTION which is applicable to system operation.

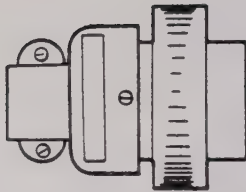
- 2) In any position of the SD1012B RANGE (Vrms) switch other than CAL, the switch is part of a closed-circuit interlock circuit via rear chassis connector J22, pins 17 and 19. Therefore, when the SD1012B RANGE (Vrms) switch is not in the CAL position, associated system instrumentation could be activated causing undesirable excitation of a specimen under test.

The following procedure assumes the use of filters having less than 100-Hz BW, with the narrowest bandwidth filter in FILTER position No. 1, Channel 1.

- a. On SD1010B, set OPER/CAL switch to CAL.
- b. On SD1012B, set both Channels 1 and 2 RANGE (Vrms) and METER MULTIPLIER switches to CAL position.
- c. On SD1012B, adjust both Channels 1 and 2 RELATIVE gain controls full clockwise to CAL position (switch detent).
- d. On SD1012B, set FILTER select switch to 1.
- e. On SD1010B, adjust the FREQ control for peak reading on SD1012B Signal Amplitude Meter of Channel 1. (If the meter peaks at full scale or over, adjust FILTER 1 CAL potentiometer for meter reading of ≈ -2 dB from full scale.)

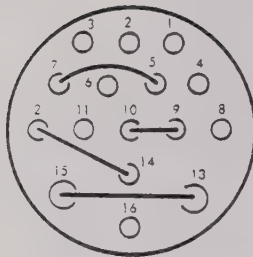
Perform the following steps f. through h. first using Channel 1 controls, switches, and meter.

- f. On SD1012B, adjust FILTER 1 CAL potentiometer for full-scale reading on Signal Amplitude Meter.
- g. If a second filter has been installed, set FILTER select switch to 2. (Do not touch FREQ control on SD1010B.)



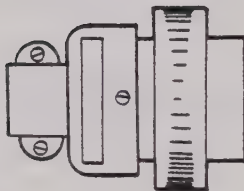
CANNON
SK-C16-21C-1/2

USE NO. 22 BUS WIRE AND CONNECT:



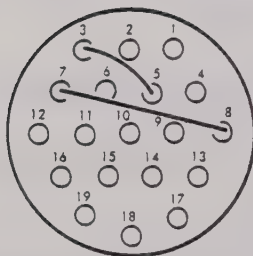
P11
(SDC 11514)

FROM	TO
5	7
9	10
12	14
13	15



CANNON
SK-19-21C-1/2

USE NO. 22 BUS WIRE AND CONNECT:



P22
(SDC 11515)

FROM	TO
3	5
7	8

Figure 2-1. Jumpered Plugs P11 and P22, Construction Details (SD1012B)

- h. On SD1012B, adjust FILTER 2 CAL potentiometer for full-scale reading on Signal Amplitude Meter.

Repeat steps f. through h. using SD1012B Channel 2 controls, switches, and meter.

2.2.3 AUTO Filter-Select Crossover Adjust

If two filters are used, automatic filter selection is preset as follows:

CAUTION

The $DC \propto$ FREQ filter-switching input voltage applied to J5 must not exceed 1 volt over the full frequency range of operation. The 0 to 1V $DC \propto$ LOG FREQ output from an SDC Model SD104A or SD114 Sweep Oscillator, or a Model SD116 Log Frequency Converter is compatible with this requirement.

- a. Connect the $dc \propto$ frequency output 1V maximum of a Sweep Oscillator (SD104A, SD116 Log Frequency Converter and a sweep oscillator, or equivalent source) to rear-chassis connector J5, CH 1-2 $DC \propto$ FREQ INPUT.
- b. On SD1012B, set FILTER switch to AUTO.
- c. On SD1012B, adjust the 1-2 AUTO crossover frequency potentiometer fully clockwise. FILTER 1 indicator lamp should be lighted.
- d. Adjust sweep oscillator to 500 Hz, FILTER 1 to 2 crossover frequency.
- e. Adjust 1-2 AUTO crossover frequency potentiometer until FILTER 2 indicator lights.
- f. Adjust sweep oscillator to sweep from approximately 100 Hz to above 500 Hz.
- g. Reset sweep oscillator to lower limit; then initiate an up-sweep. Observe the sweep oscillator frequency meter reading and the FILTER indicators; crossover should occur between FILTER 1 and 2 at 500 Hz.

2.2.4 SINE REJECT (Models SD1012B-1 and SD1012B-2 only)

The Model SD1012B-1 contains Sine Reject Card A3 in Channel 1; the Model SD1012B-2 contains Sine Reject Cards A3 and A7 in Channels 1 and 2 respectively. After the filter(s) have been calibrated in accordance with paragraph 2.2.2, check for proper sine reject operation as follows:

- a. Set FILTER switch to 1, RANGE switch to 1, and METER MULTIPLIER switch to 1.

- b. Apply 400-Hz, 1 Vrms (full-scale) signal to both the SD1012B SIGNAL INPUT (J6) and to the SD1010B TUNING INPUT (J1).
- c. On SD1010B, adjust front-panel TUNING AMPL control for center-line (green area) reading on TUNING AMPLITUDE meter.
- d. Connect an rms voltmeter (HP3400A or equivalent) to J9, SINE REJECT OUTPUT.
- e. Adjust front-panel CHANNEL 1, FILTER 1 ϕ adjust and the internal 180° phase reversal switch for minimum rms reading at J9.
- f. Now alternately adjust the CHANNEL 1, FILTER 1 ϕ adjust and the CHANNEL 1 SINE REJECT NULL for minimum output, at least 30 dB below 1 Vrms.

NOTE

If SINE REJECT output cannot be properly reduced using the CHANNEL 1, FILTER 1 ϕ adjust and SINE REJECT NULL, adjust very slightly the FREQ adjust on the SD1010B Carrier Generator.

CAUTION

A PSD Plug-in Assembly (11312-) MUST BE installed in each Crystal Filter Channel used or the instrument WILL NOT operate in PSD mode. Refer to paragraph 2.2.1 for installation NOTES.

- g. If instrument is Model SD1012B-2, repeat steps a. through f. using J14 (SIGNAL INPUT), J21 (SINE REJECT OUTPUT), and Channel 2 ϕ adjust and SINE REJECT NULL. (Do not touch FREQ adjust on SD1010B.)
- h. Proceed with the final operational checks in the following paragraphs.

2.2.5 Final Operational Checks

After the filters have been calibrated and crossover adjustment checked, check for proper outputs from each channel as follows:

- a. Set FILTER switch to 1, both RANGE (Vrms) switches to 1, both METER MULTIPLIER switches to 1, and MODE switch to PSD 1.
- b. Apply 400-Hz, 1 Vrms (full-scale) signal to both the SD1012B SIGNAL INPUT (J6) and to the SD1010B TUNING INPUT (J1).

- c. On SD1010B, adjust front-panel TUNING AMPL control for center-line (green area) reading on TUNING AMPLITUDE meter.
- d. Using applicable test equipment, check for the following outputs:

<u>Check Point (CH 1, CH 2)</u>	<u>Reading</u>
FIL SIG UNITY GAIN (J4, J16)	1 Vrms $\pm$ 40 mV rms, 400 Hz
FIL SIG 10V F.S. (J3, J17)	10 Vrms $\pm$ 0.4 Vrms, 400 Hz
100-kHz FIL I.F. (J7, J19)	1.8 Vrms $\pm$ 80 mV rms
LOG DC (J8, J18) (PSD MODE)	400 mV $\pm$ 5 mV
LIN DC (J10, J20)	+10.0V $\pm$ 50 mV
Signal Amplitude Meters	1 Vrms full-scale

Remove all test equipment, remove power, and place instrument in service.

2.3 OPERATION

2.3.1 General

The following paragraphs describe the operation of the SD1012B (SD1012B-1, -2) in various applications. In each case, certain preliminary adjustments are required. These are described in paragraphs 2.2.1 Preliminary Setup, 2.2.2 Filter Calibration, 2.2.3 AUTO Filter-Select Crossover Adjust (using desired frequencies), and 2.2.4 Sine Reject (for Models SD1012B-1, -2).

2.3.2 Use as a Dynamic Analyzer/Tracking Filter

- a. Perform the applicable preliminary adjustments of paragraphs 2.2.1 through 2.2.4.
- b. Set the front-panel RANGE (Vrms) and METER MULTIPLIER switches as required by the expected dynamics of the SIGNAL INPUT (both channels).
- c. Connect the 2.4V p-p carrier output of the associated SD1010B (or SD101B) to J15, CH 1-2 CARRIER INPUT.
- d. Apply the system tuning signal (constant-level sine wave) to the TUNING INPUT of the associated SD1010B (or SD101B).
- e. On SD1010B (or SD101B), adjust the front-panel TUNING INPUT AMPL adjust for center-scale (green area) reading on TUNING AMPLITUDE meter.
- f. Set the MODE switch to the desired SINE AVG (SEC) position.

Table 2-1. AUX FUNCTIONS Connector J11 Circuit Descriptions

J11 Pin	Function Description
1	Sweep rate control ground return circuit
2	DC $\propto$ frequency output for slaving remote filter select
3	100-kHz I.F. unfiltered output (Channel 1)
4	Shield ground for pin 3
* 5	100-kHz FIL I.F. input (return from pin 7 output or external filter)
6	Shield ground for pins 5 and 7
* 7	100-kHz FIL I.F. output, (Channel 1, FIL 2)
8	External d-c input (Channel 1) for averaging
* 9	Averaged d-c output (Channel 1, Filter 2)
*10	DC averaging filter BW control
11	Spare
*12	Remote sweep rate control source (Channel 1, Filter 2 position only)
*13	External filter position indicator control ground return for pin 15
*14	Sweep rate control output (Channel 1, Filter 2)
*15	External filter position indicator control
16	Slaving remote BW select

*See Figure 2-1 for P11 plug wiring details.

Table 2-2. AUX FUNCTIONS Connector J22 Circuit Descriptions

J22 Pin	Function Description
1	100-kHz I.F. unfiltered output (Channel 2)
2	Shield ground for pin 1
* 3	100-kHz FIL I.F. output (Channel 2, FIL 2)
4	Shield ground for pins 3 and 5
* 5	100-kHz FIL I.F. input (return from pin 3)
6	External d-c input (Channel 2) for averaging
* 7	Averaged d-c output (Channel 2, Filter 2)
* 8	DC averaging filter BW control
9	Slaving control (+dc in Filter 2 position)
10	Sweep rate output (to SD104A J30-J, or SD114 J9-8)
11	+15V or 15V reference input from Sweep Oscillator (SD104A J30-L, or SD114 J9-4)
12	Return path for internal oscillator sweep rate control (from SD104A J30-H, or SD114 J9-9)
13–16	Spares
17	System closed-loop interlock circuit via front-panel RANGE switches (See also pin 19.)
18	Sweep rate control ground return circuit
19	Interlock circuit return for pin 17

*See Figure 2-1 for P22 plug wiring details.

NOTE

When analyzing complex sinusoidal signals, and d-c averaging is used, the time indicated on the MODE switch is the d-c averaging-circuit time constant (for the LOG and LIN DC outputs). To determine the overall instrument time constant, the selected filter bandwidth must be considered.

- g. Connect desired external recording equipment to the output connectors. All outputs of the SD1012B will be proportional to the SIGNAL INPUT (in the same channel) at the tuned frequency as described in Section III of this manual.

NOTE

If the FILTER switch is set to OUT, the UNITY GAIN FIL SIG OUTPUTS (J4, J16) and FILTER OUTPUT (front panel) connectors are switched directly to the SIGNAL INPUT. However, the SIGNAL is processed by the instrument (FILTER AUTO-select mode) and all other outputs are normal, including the Signal Amplitude meter readings, FIL SIG 10V F.S., LOG DC, LIN DC, etc.

2.3.3 Use as a Power Spectrum Analyzer (PSD Analysis)

CAUTION

A PSD Plug-in Assembly (11312-) MUST BE installed in each Crystal Filter channel used or the instrument WILL NOT operate in PSD mode. Refer to paragraph 2.2.1 for installation NOTE.

NOTE

The standard PSD Plug-in Assemblies (11312) for the SD1012B are designed for use over the frequency range of 5 Hz to 5 kHz; for example, a Model SD104A-5 Sweep Oscillator with its MULTIPLIER (A) switch set to 1, or a Model SD114 Sweep Oscillator Servo. The

(Cont'd)

oscillators then have a maximum sweep rate of 100 Hz/s and a sweep rate sensitivity of 0.1 V/Hz/s. If other frequency ranges are desired using the SD104A series of sweep oscillators, refer to Appendix A (rear of this manual) for calculations of proper sweep rate resistor values.

For optimum power spectrum analysis, the SD1012B should be used with a Model SD1010B Carrier Generator, a Model SD104A Sweep Oscillator or Model SD114 Sweep Oscillator Servo, and an X-Y Recorder. (See Figure 2-2.)

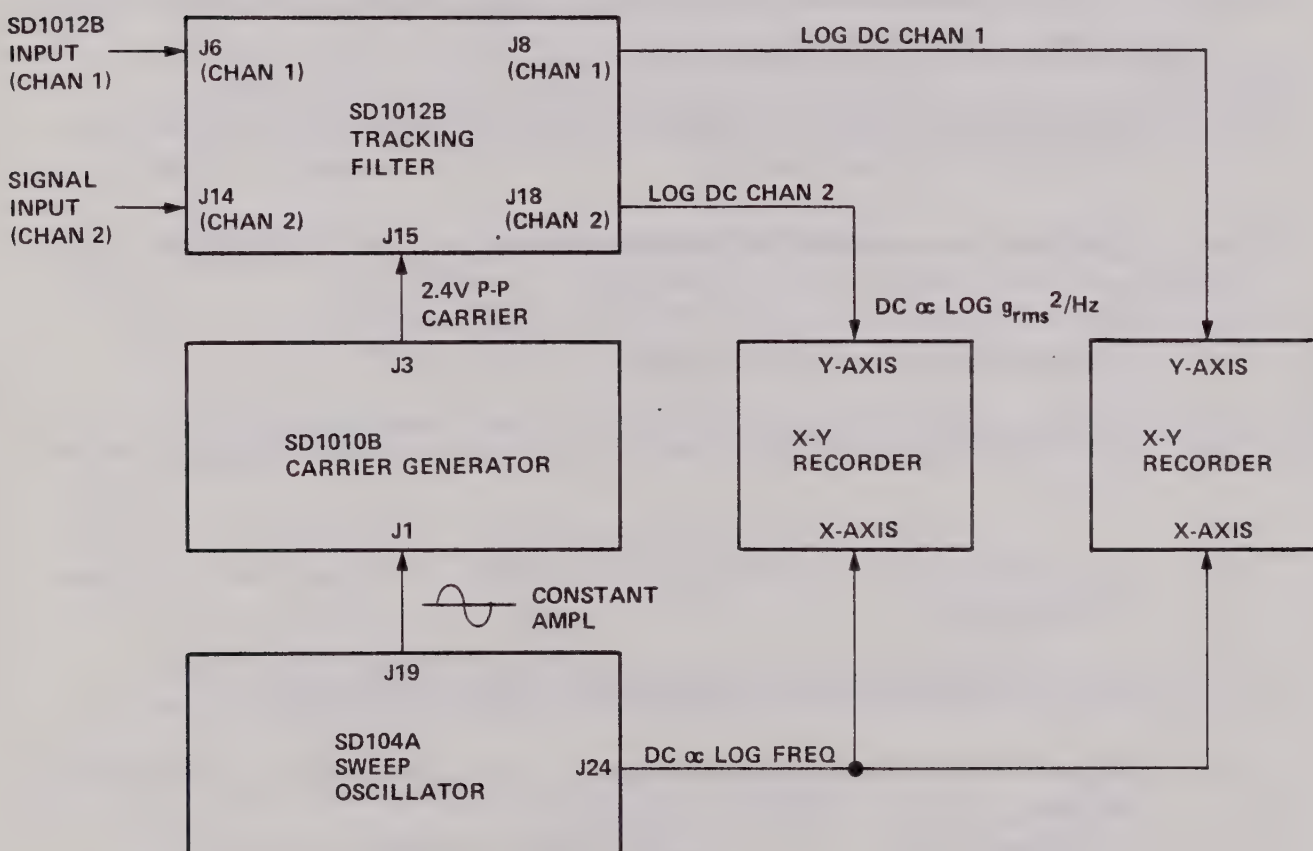


Figure 2-2. Equipment Setup for PSD Analysis

Using the setup of Figure 2-2, the SD1012B can be calibrated in either of two ways: The internal calibration signal of the SD1012B representing a known absolute voltage level may be used, or an external calibration signal representing a certain known physical quantity such as 10g rms may be applied to the input of the SD1012B from an on-line test or from a tape recording. In either case,

a compatible carrier signal (100 kHz + tuning frequency) must be applied to the SD1012B from an SD1010B Carrier Generator. Figure 2-2 shows 2-channel operation using two X-Y recorders; however, either channel may be operated separately.

2.3.3.1 Calibration Using Internal SD1012B and SD1010B CAL Signal

Use the following procedure to calibrate Channel 1; then if both channels are used, calibrate Channel 2 in like manner.

- a. Connect the SD1012B, SD1010B, SD104A, and X-Y recorder(s) as shown in Figure 2-2. Power all equipment and allow at least 1 hour for warmup. It is assumed that two crystal filters and their associated PSD assemblies have been installed in accordance with the procedural NOTES of paragraph 2.2.1.
- b. On SD1012B, set RANGE (Vrms) switch to CAL; adjust RELATIVE control to CAL (fully clockwise in switch detent).
- c. On SD1012B, set METER MULTIPLIER switch to CAL and set FILTER select switch to 1.
- d. On SD1010B, set OPER/CAL switch to CAL.
- e. On SD1010B, adjust the FREQ control for peak reading on SD1012B Signal Amplitude Meter.
- f. On SD1012B, adjust FILTER 1 CAL potentiometer for full-scale reading on Signal Amplitude Meter.
- g. Set FILTER select switch to 2. (Do not touch SD1010B FREQ control.)
- h. Adjust FILTER 2 CAL potentiometer for a meter reading based on the following equation:

$$\text{FILTER 2 CAL meter reading} = \text{Full Scale} \times \sqrt{\frac{\text{BW1}}{\text{BW2}}}$$

For example: If Filter 1 BW is 5 Hz and Filter 2 BW is 10 Hz, FILTER 2 CAL potentiometer is adjusted for a meter reading of 0.707 of full scale ($\sqrt{0.5 \times 1}$).

- i. See Table 2-3 and set the SD1012B MODE switch to the desired PSD CONFIDENCE position (1, 2, 3 or 4).
- j. On the SD104A, preset the desired upper and lower sweep limits. Adjust the SD104A for LINEAR sweep mode.
- k. Calibrate the X-axis of the X-Y recorder to the desired frequency sweep range (step j. above). This procedure assumes the use of log/log graph paper. Alternately press RESET UP and RESET DOWN while adjusting the recorder ZERO and GAIN control until the upper and lower limits line up with the corresponding frequencies on the graph paper.
- l. Calibrate the Y-axis of the X-Y recorder by stepping the SD1012B DC ATTEN (dB) switch in 10 dB steps; make each 10 dB step correspond to 1 decade on the graph paper.

Up to this point, the relative span of both the X and Y axes have been correctly calibrated. Now, an absolute number must be assigned in terms of, for example, g_{rms}^2/Hz for a point on the Y-axis of the X-Y recording. Also, a full-scale range setting must be established which will be used during analysis.

For example: If a random vibration test of $20 g_{rms}$ is to be performed and a signal sensitivity of $10 \text{ mV rms}/g_{rms}$ is employed, a broadband signal of approximately 200 mV rms will be applied to the SD1012B. For this example, the SD1012B range (V_{rms}) setting is 0.3; thus, full-scale meter reading corresponds to $31.6 g_{rms}$.

The Y-axis calibration point on the recorder is now established using the equation: (See footnote.)

$$CAL = \frac{(\text{Full Scale} \times 1.13)^2}{BW}$$

If a 5 Hz bandwidth filter were being used in the first filter position, the calibration point for the above example would become:

$$CAL = \frac{(31.6 g_{rms} \times 1.13)^2}{5 \text{ Hz}} = 255 g_{rms}^2/\text{Hz}$$

Now adjust the Y-axis zero control of the X-Y recorder so the pen lines up exactly on a reading of 255 on the top decade of the log/log paper. Because this number is an extremely high PSD

*Additional technical information concerning PSD analysis is included in SDC Technical Publications PS-1 and PS-2 in the back of this manual.

Table 2-3. PSD Confidence Parameters

Filter BW	PSD Confidence Switch Position			
	1	2	3	4
1.5	30	30	40	40
2	40	40	40	40
5	40	80	100	100
10	40	80	120	200
20	40	80	120	200
50	40	80	120	200
100	80	80	120	200
200	80	120	160	240
1.5	5	5	6.67	6.67
2	5	5	5	5
5	2	4	5	5
10	1	2	3	5
20	0.5	1	1.5	2.5
50	0.2	0.4	0.6	1
100	0.2	0.2	0.3	0.5
200	0.1	0.15	0.2	0.3
1.5	* 0.1 (0.075)	* 0.1 (0.075)	* 0.1 (0.055)	* 0.1 (0.055)
2	0.1	0.1	0.1	0.1
5	0.625	0.313	0.25	0.25
10	2.5	1.25	0.83	0.5
20	10	5	3.33	2
50	62.5	31.3	21	12.5
100	*100 (125)	*100 (125)	83	50
200	*100 (500)	*100 (333)	*100 (250)	*100 (166)

*The values listed are within the sweep rate programming range of the SD104A-2 and SD104A-5 (0.1 to 100 Hz per second); the values in parenthesis are the calculated values. If an SD104A-1 or an SD104-10 is used, consult your SDC Factory Representative for correct PSD Assembly information.

level and will in all actual cases never be achieved during a test, a level more consistent with the expected data such as $0.255 \text{ g}_{\text{rms}}^2/\text{Hz}$ can now be set up on the recorder by attenuating the log dc output three decades in three 10 dB steps. (Recall that a 10 dB change on the DC ATTEN (dB) has been calibrated to equal one decade on the X-Y recorder paper.) If the log dc output is attenuated by three 10 dB steps, the calibration point on the recorder becomes $0.255 \text{ g}_{\text{rms}}^2/\text{Hz}$. After the 30 dB of attenuation has been introduced, adjust the X-Y recorder ZERO control again for an exact reading of 255 on the top decade of the log/log graph paper. The top of the graph paper now represents $1.0 \text{ g}_{\text{rms}}^2/\text{Hz}$.

After the frequency and amplitude axes of the recorder have been calibrated, adjust the filter crossover frequencies as follows:

- m. On SD1012B, set FILTER switch to AUTO.
- n. On SD1012B, adjust the 1-2 AUTO crossover frequency potentiometer fully clockwise. (FILTER 1 indicator should be lighted.)
- o. Adjust sweep oscillator to desired FILTER 1 to 2 crossover frequency.
- p. Adjust 1-2 AUTO crossover frequency potentiometer until FILTER 2 indicator lights.

After performing the preceding steps, a PSD analysis is performed as follows:

- q. On SD104A rear chassis, place the SWEEP RATE switch to EXT. (Ensure that the correct interconnect cable is connected between J30 of the SD104A and J11 and J22 of the SD1012B.)
- r. On SD104A, set the sweep mode switch to SINGLE LINEAR SWEEP.
- s. On SD1012B, set MODE switch to desired PSD CONFIDENCE position (1, 2, 3 or 4). (See Table 2-3.)
- t. On SD1012B, set LOG DC ATTEN (dB) switch to (OPER) 0.
- u. On SD1010B, set OPER/CAL switch to OPER.
- v. Apply signal to be analyzed (output of accelerometer) to SD1012B SIGNAL INPUT.
- w. Start test by pressing the SD104A SWEEP UP push button.

2.3.3.2 Calibration Using an External Calibration Signal

To calibrate to an external signal, the relative span of the recorder's X and Y axes are first calibrated to the SD1012B and SD1010B internal CAL signal. The X-axis is calibrated to the frequency sweep range, and then the Y-axis is calibrated to the external signal which is applied to the SD1012B SIGNAL INPUT. Proceed as follows:

- a. Perform steps a. through l. of paragraph 2.3.3.1.
- b. Refer to Figure 2-2, and apply the external calibration signal (sinusoidal) to the SD1012B SIGNAL INPUT.
- c. On SD1012B, select widest BW filter which will be used for the subsequent PSD analysis. Note, however, that the filter BW must not exceed two times the calibration frequency.
- d. On SD1012B, set the RANGE (V_{rms}) switch (with signal applied) so that the Signal Amplitude Meter reads within the upper 10 dB of scale selected. (The OVERLOAD indicator must not be lighted.)
- e. Set the SD104A sweep mode switch to MAN., and manually adjust its output to equal the calibration frequency. This is accomplished by adjusting the SD104A frequency for a peak reading on the SD1012B Signal Amplitude Meter.
- f. Compute the values of the calibration parameters as follows:

$$\text{Calibration Value} = \frac{(\text{External cal sig in } g_{rms} \times 1.13)^2}{\text{BW in Hz}}$$

For example: If the external calibration signal input represents 8 g_{rms} , and a 20-Hz BW filter is used, then:

$$\text{CAL Value} = \frac{(8 g_{rms} \times 1.13)^2}{20} = 4.07 g_{rms}^2/\text{Hz}$$

NOTE

Assuming the preceding example, after the SD1012B is properly tuned (peak reading on meter) and calibrated, the meter reading truly represents 8 g_{rms} corresponding to a resultant PSD calibration point of 4.07 g_{rms}^2/Hz . (The SD1012B Signal Amplitude Meter will not necessarily read 8.)

- g. Adjust the Y-axis zero control of the X-Y recorder so the pen lines up exactly on a reading of 4.09 on the top decade of the log/log paper. Because this number may be quite high and may not be achieved during a test, a level more consistent with the expected data such as $0.04 \text{ g}_{\text{rms}}^2/\text{Hz}$ can now be set up on the recorder by attenuating the log dc output two decades in two 10 dB steps. (Recall that a 10 dB change on the log dc attenuator has been calibrated to be equal to one decade on the X-Y recorder paper.) If the log dc output is attenuated by two 10 dB steps, the calibration point on the recorder now becomes $0.04 \text{ g}_{\text{rms}}^2/\text{Hz}$. After the 20 dB of attenuation has been introduced, adjust the X-Y recorder ZERO control again for an exact reading of 4.09 on the top decade of the log/log graph paper.

After the frequency (X) and amplitude (Y) axes of the recorder have been calibrated, adjust the filter crossover frequencies as follows:

- h. On SD1012B, set FILTER switch to AUTO.
- i. On SD1012B, adjust the 1-2 AUTO crossover frequency potentiometer fully clockwise. FILTER 1 indicator lamp should be lighted.
- j. Adjust sweep oscillator to desired FILTER 1 to 2 crossover frequency.
- k. Adjust 1-2 AUTO crossover frequency potentiometer until FILTER 2 indicator lights.

After performing the preceding steps, the PSD analysis is performed as follows:

- l. On SD104A rear chassis, place the SWEEP RATE switch to EXT. (Ensure that the correct interconnect cable is connected between J30 of the SD104A and J11 and J22 of the SD1012B.
- m. On SD104A, set the sweep mode switch to SINGLE LINEAR SWEEP.
- n. On SD1012B, set MODE switch to desired PSD CONFIDENCE position (1, 2, 3 or 4).
- o. On SD1012B, set LOG DC ATTEN (dB) switch to (OPER) 0.
- p. On SD1010B, set OPER/CAL switch to CAL.
- q. Apply signal to be analyzed (output of accelerometer) to SD1012B SIGNAL INPUT.

NOTE

If the SD1012B RANGE (V_{rms}) switch must be changed to accommodate the level of data signal being analyzed, recall that a 20 dB change in the input range is equivalent to a 2 decade change in the plotted power spectrum.

- r. Start test by pressing the SD104A SWEEP UP push button.

2.3.4 Using SINE REJECT (J9/J21) OUTPUT (Model SD1012B-1/SD1012B-2)

The SD1012B-1 contains in Channel 1 only an additional circuit card A3, Sine Reject Assembly 10954-1. The Model SD1012B-2 contains cards A3 and A7 (identical cards) which provide SINE REJECT OUTPUTS in both Channels 1 and 2. (See schematic DWG. NO. 11386, Sheet 1 of 4, at rear of manual.) The following simplified block diagram (Figure 2-3) shows how the raw input data is routed to the sine reject circuit. Here it is mixed with the filtered signal output (displaced in phase by 180 degrees), and the data signal at the tuning frequency is thus canceled, leaving all other data components in the SINE REJECT OUTPUT.

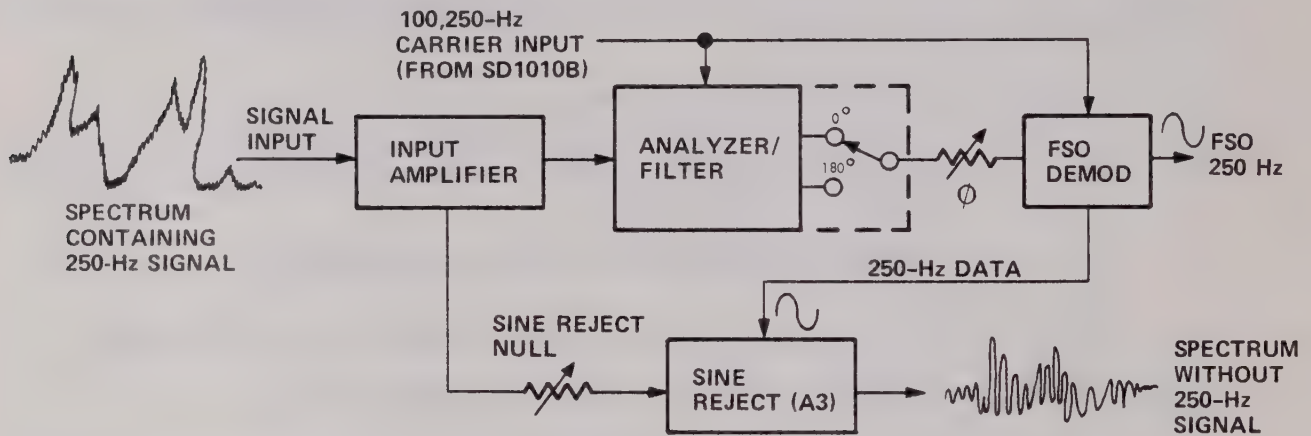


Figure 2-3. Simplified Block Diagram of Sine Reject Circuit (Channel 1)

To use a Model SD1012B-1 (or -2) for sine reject operation, interconnect the unit and all other applicable instrumentation in accordance with the particular test requirements. Then proceed as follows:

- Power the equipment, apply proper carrier signal, calibrate the filter(s), etc. as described in paragraphs 2.2.1 through 2.2.3.
- Perform the procedure of paragraph 2.2.4 using any frequency within the operating range of the test (2 Hz to 50 kHz).

CAUTION

To ensure proper operation, the SINE REJECT circuits should always be operationally checked immediately before a system operation.

SECTION III

CONCEPT AND THEORY OF OPERATION

3.1 CONCEPT OF OPERATION

The Model SD1012B Tracking Filter utilizes balanced modulators and a 100-kHz (plus tuning frequency) carrier signal in a patented heterodyne technique to select and translate the fundamental frequency (2 Hz to 50 kHz) from a complex spectrum into a phase and amplitude coherent 100-kHz signal. The unfiltered 100-kHz I.F. signal is filtered at selected bandwidths and then demodulated to produce the fundamental frequency (FSO — filtered signal output). Detection of the FSO and subsequent log conversion provide both linear and log d-c voltage outputs that are proportional to the input signal at the tuned frequency.

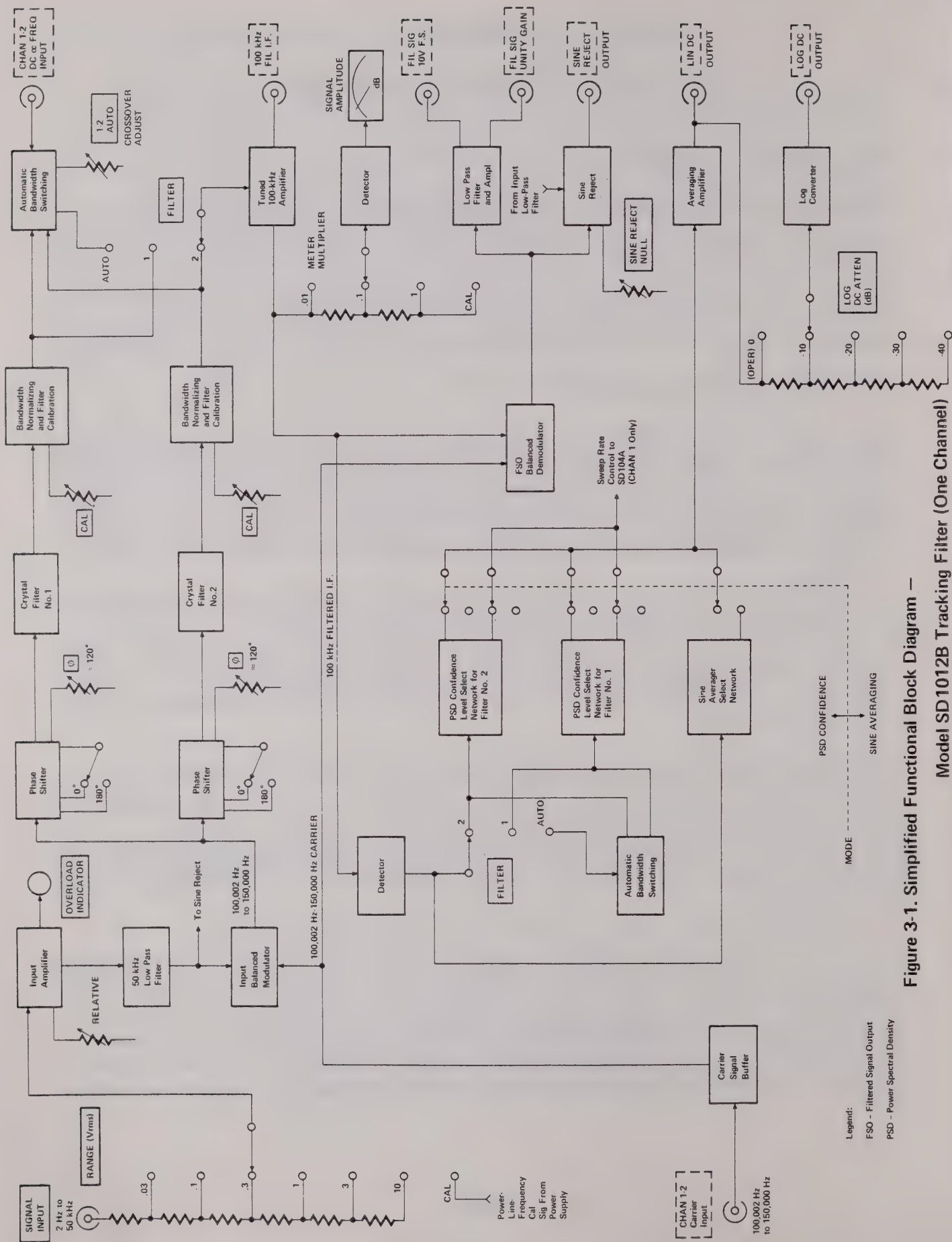
3.2 BLOCK DIAGRAM DESCRIPTION

Both channels of the SD1012B operate the same; therefore, only one channel is described here and shown in the block diagram of Figure 3-1. The following description assumes a complex SIGNAL INPUT containing a component at 250 Hz. The CARRIER INPUT signal is 100,250 Hz. The OVERLOAD indicator is not lighted. (If lighted, it indicates an overload condition exists; i.e., the input signal is 12 dB over full scale of the RANGE selected, and that an advancement to the next highest range is required.)

The data SIGNAL INPUT being analyzed is routed through the RANGE (V_{rms}) switch (step attenuator) to the input amplifier. The amplified signal is filtered (low pass) and applied to one input of the input balanced modulator. At the same time, the 100,250-Hz CARRIER INPUT signal is buffered and applied to the other input of the balanced modulator. The input balanced modulator translates the data signal to a phase and amplitude coherent 100-kHz intermediate frequency (I.F.) signal. The unfiltered I.F. signal is then filtered and amplified.

Each of the two filter circuits comprises three main parts: phase shifter, crystal filter, and bandwidth normalizing and filter calibration. The phase shifter provides a means for manually normalizing the input-to-output phase of the signal through each filter relative to each other, or normalizing the output phase relative to that of another unit such as an SD101B Dynamic Analyzer or another SD1012B. Front panel ϕ control provides $\approx 120^\circ$ variable phase shift in addition to 0° or 180° phase shift that is switch selected internally.

Each filter is a plug-in type, multisection crystal lattice filter, and is contained in a temperature-controlled oven. The bandwidth normalizing and filter calibration circuits in each channel facilitate normalizing the 100-kHz I.F. output signal level over the two selectable bandwidths. The outputs of the two filter channels can be selected manually or automatically. Automatic selection is controlled by a d-c voltage proportional to the tuning frequency. (This voltage (1 volt maximum) is normally supplied by the system sweep oscillator, i.e., a Model SD104A, or from a log frequency converter, i.e., a Model SD116.)



The 100-kHz I.F. output from the selected filter is amplified and provided as an output at the 100-kHz FIL I.F. rear chassis connector for use in phase measurement and other applications. The amplitude of the 100-kHz filtered I.F. signal (which is equal to the amplitude of the SIGNAL INPUT at the tuning frequency) is continuously displayed on the front panel Signal Amplitude meter in both volts (rms) and relative dB level.

The 100-kHz I.F. is further processed to develop four types of outputs: a filtered signal output (FSO) at the tuned frequency, a sine reject output, and two d-c voltage outputs, a linear dc and a log dc. All outputs (except the sine reject) are proportional to the SIGNAL INPUT (at the tuned frequency). These outputs are developed as follows:

The 100-kHz I.F. signal is combined with the carrier signal (100,250 Hz) in the FSO balanced demodulator. The FSO demodulator translates the 100-kHz filtered I.F. to a 250-Hz signal which is phase coherent with the 250-Hz signal contained in the input spectrum. The 250-Hz signal is filtered and amplified, and provided at two output levels: the FSO OUTPUT 10V F.S. provides 0 to 10V full scale over any RANGE selected; the FSO OUTPUT UNITY GAIN provides the filtered data signal at the applied INPUT SIGNAL level.

*The output of the FSO demodulator is also applied to the sine reject circuit. The SINE REJECT output signal is employed during combined sine/random testing; i.e., when both sinusoidal and random noise signals are used to excite a specimen. The SINE REJECT OUTPUT provides all accelerometer signals except the tuned frequency itself. That is, the tuned frequency (FSO), which normally exhibits the largest amplitude, is suppressed; the random signals are passed through the tracking filter at unity gain. The random signals from the SINE REJECT OUTPUT can now be further analyzed by an external filter-type analyzer/controller. The external analyzer/controller can be operated in the automatic mode without possibility of over-compression, a condition that would occur if the large amplitude tuned frequency signal were present.

The linear and log d-c voltage outputs are characterized by the type of input signal and the mode of operation; the d-c averaging time constant of the d-c outputs is a function of the MODE switch setting. Assuming the analyses of periodic data, the MODE switch is set to the desired averaging time position, SINE AVG (SEC). In this mode, a d-c voltage proportional to the data amplitude is detected from the 100-kHz I.F., filtered and averaged in the selected sine averager network, and applied to the averaging amplifier. Assuming stationary random signal inputs, the MODE switch is set to PSD CONFIDENCE (position 1, 2, 3 or 4 as desired). In this mode, the d-c output of the detector is filtered and averaged in the selected PSD confidence level select network and then applied to the averaging amplifier. Note that the proper PSD network for each filter bandwidth is selected manually or automatically as a function of the FILTER switch setting, thus giving proper averaging time constants consistent with PSD measuring techniques.

*This circuit is optional and is included in Channel 1 of Model SD1012B-1 and in Channels 1 and 2 of Model SD1012B-2.

In either mode, the averaged d-c voltage is amplified and provided at the LIN DC OUTPUT connector. This output is linearly proportional to the amplitude of the data input signal (at the tuned frequency).

The linear dc is also applied to a log converter. This circuit provides a d-c output voltage proportional to the log of the linear d-c voltage, hence, proportional to the log of the amplitude of the input signal (at the tuned frequency). Note that when switching from the SINE AVERAGING MODE to the PSD CONFIDENCE MODE, the LOG DC output is doubled.

SECTION IV

CALIBRATION AND MAINTENANCE

4.1 GENERAL

Before calibrating the instrument, the technician is advised to study carefully Section III of this manual. Refer also to the front-panel and rear-chassis descriptions given in Section I.

Most voltage checks and adjustments can be made with the circuit cards installed. Remove and extend cards only when directed, and then use caution when installing extender and circuit card; be sure to remove power. Figure 4-1 is a top view of the instrument with cover removed, showing all calibration controls.

Note that except for the extender card, each circuit card is individually keyed to its receptacle. Each card is slotted, and a "key" is located between two pins of each receptacle. Therefore, the "key" must be removed before the extender can be inserted.

NOTE

A layout drawing showing the location of the circuit cards is located on the inside surface of the top cover. On this drawing, the location of each circuit card "key" is denoted by its two adjacent pin numbers. For example, on circuit card A1, the "key" is located between pins 13 and 14.

CAUTION

To prevent possibility of plugging a circuit card in the wrong socket, be sure to replace "keys" after calibrating or servicing the instrument.

Throughout the calibration procedure, calibration controls and parts on a circuit card are referenced by their circuit card and part number alphanumeric designator. For example, on Power Supply circuit card A9, R15 is referred to as A9R15. Similarly, circuit card pin connections are referenced by the circuit card alphanumeric designator, a hyphen, pin number/letter; such as A2-F, or A4-16.

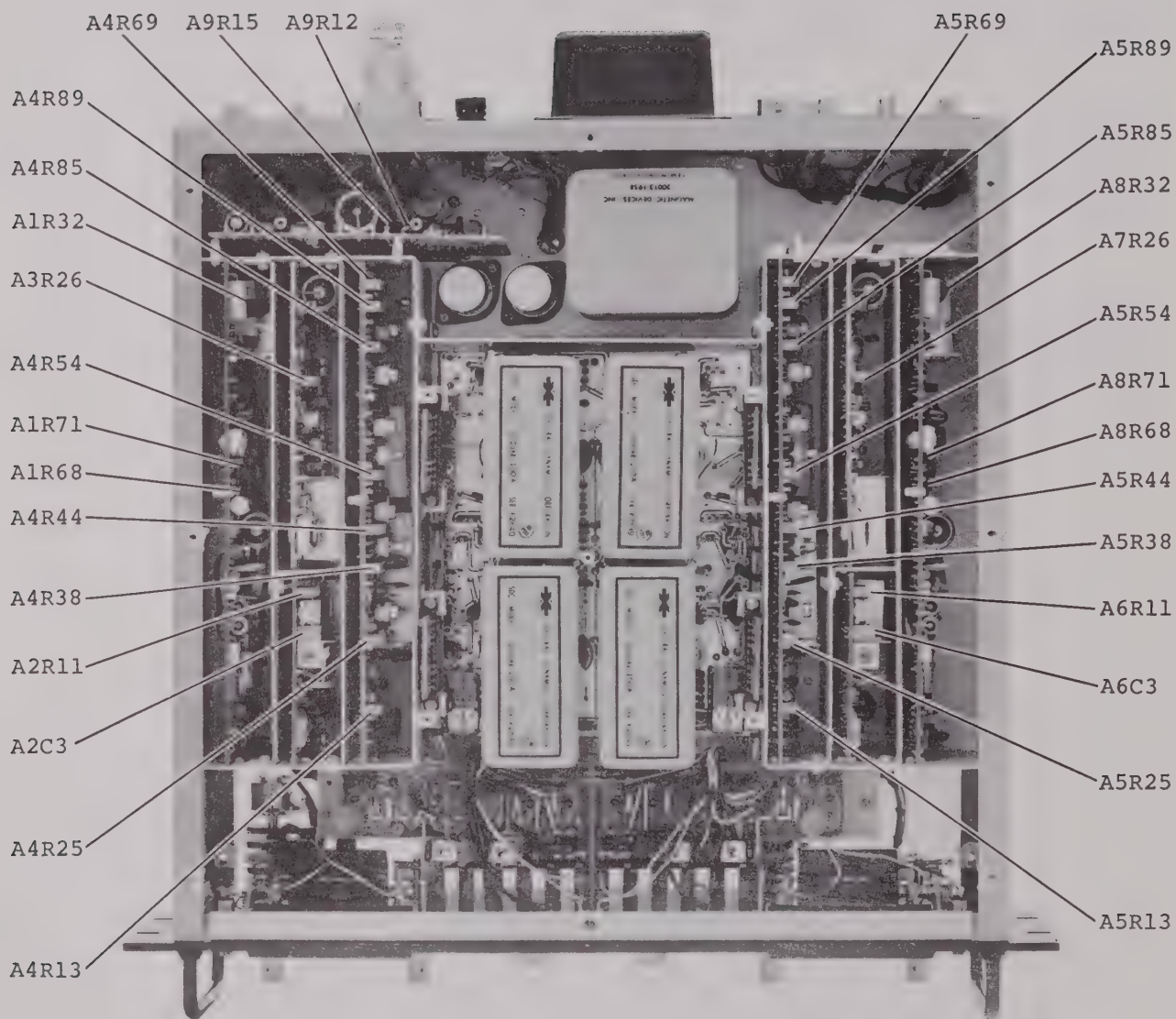


Figure 4-1. Top Chassis View

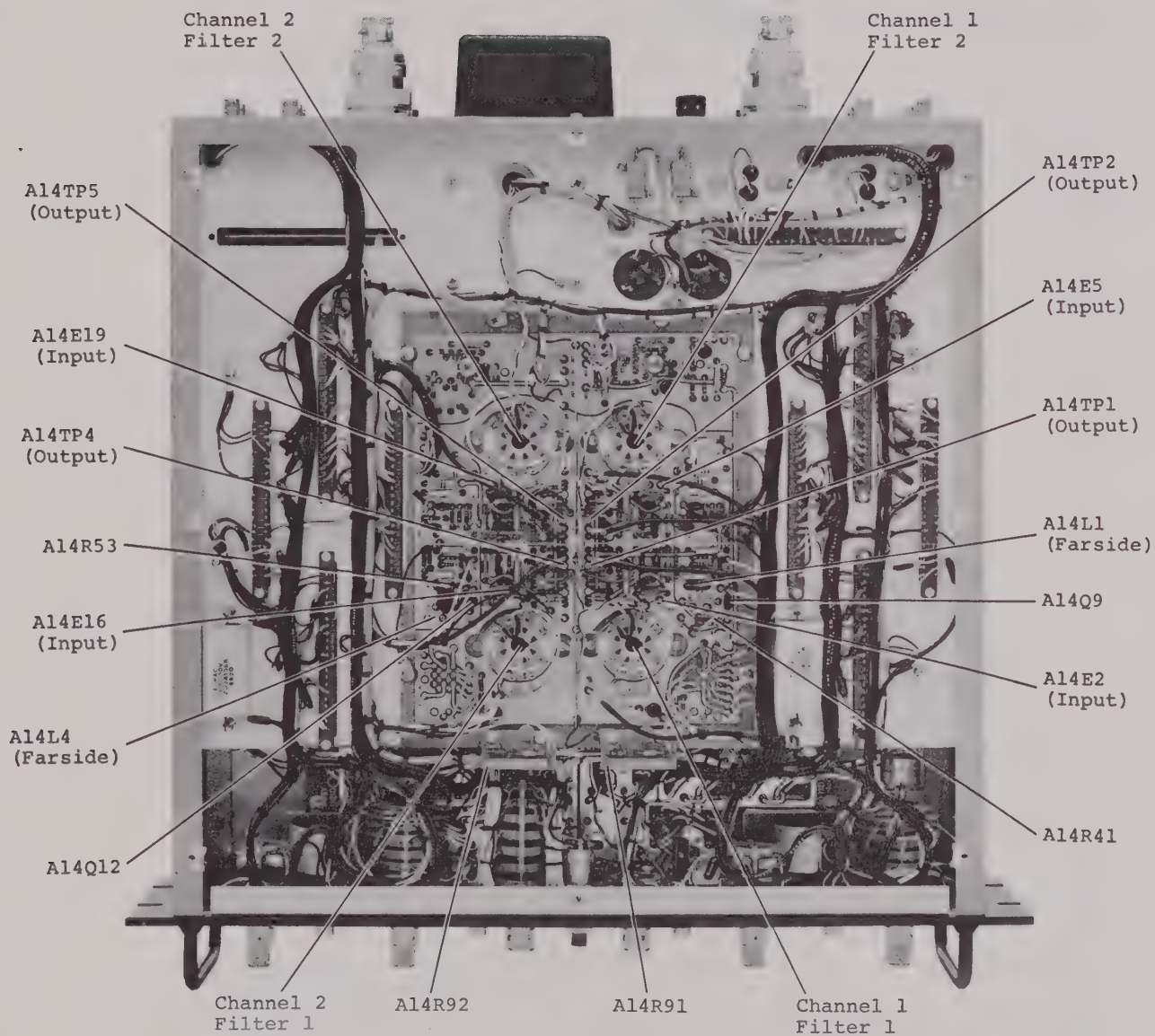


Figure 4-2. Bottom Chassis View

4.2 CALIBRATION PROCEDURE

CAUTIONS

- 1) To ensure proper calibration of the instrument, the following procedure must be performed in its entirety and in the sequence listed.
- 2) Be sure power is off before removing or replacing circuit cards.

Table 4-1 lists the test equipment required; test equipment having equal or better operational characteristics may be substituted.

Table 4-1. Test Equipment Required

Name	Model No.	Manufacturer
Attenuator	350D	Hewlett-Packard
Digital Voltmeter	3440A	Hewlett-Packard
Carrier Generator	SD1010B	Spectral Dynamics
Frequency Counter	5223L	Hewlett-Packard
RMS Voltmeter	3400A	Hewlett-Packard
Ratio Transformer	RT-60	Gertsch
Sweep Oscillator	SD104A	Spectral Dynamics
Oscilloscope	545	Tektronix
Probe	013-071	Tektronix
Wave Analyzer	310	Hewlett-Packard

4.2.1 Preliminary Setup

- a. With the power off, check that the front-panel Signal Input meters read 0. Mechanically adjust if necessary.
- b. Check that the 115/230 switch on the rear chassis is set properly and the correct value of fuse is installed.
- c. Ensure that jumpered plugs P11 and P22 are installed. (See Figure 2-1 in Section II for plug details.)
- d. Remove the instrument top cover and ensure installation of at least one crystal filter in each channel. (Refer to paragraph 2.2.1 NOTES in Section II.)

NOTE

If available, install 5-Hz and 50-Hz BW filters in positions 1 and 2 respectively of each channel. Other BW values can be used, such as 2 Hz, 10 Hz, etc. If only one filter is available, install it in position 1 and ensure that its PSD assembly is also installed in PSD No. 1 position.

- e. Connect power cord to the primary power source, and press the front-panel POWER switch/indicator; it should glow red indicating power is on.

NOTE

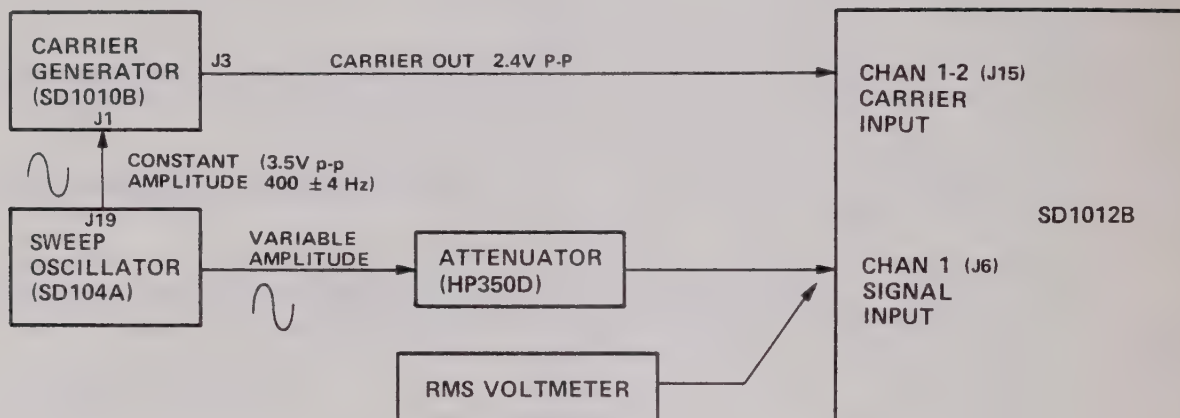
A 5-minute warmup is sufficient before making the power supply adjustments. However, at least a 1-hour warmup should be allowed before calibrating the filter section.

4.2.2 Power Supply (A9)

- a. Connect DVM and oscilloscope to A9TP4. Adjust A9R15 for -15V (-14.98 to -15.02). Ripple should be less than 4 mV p-p.
- b. Connect DVM and oscilloscope to A9TP3. Adjust A9R12 for +15V (+14.98 to +15.02). Ripple should be less than 4 mV p-p.
- c. Connect DVM and oscilloscope to A9TP9. DVM should read approximately +45V; ripple should be less than 5 mV p-p.

4.2.3 AC INPUT/OUTPUT (A1), Input Section (Channel 1)

Connect the SD1012B and test equipment as follows:



- On the SD1010B, set the OPER/CAL switch in the OPER position and adjust the TUNING AMPL control for a centerline indication (green area) on the TUNING AMPLITUDE meter.
- On Channel 1 of the SD1012B, set the RANGE (Vrms) and METER MULTIPLIER controls in the 1 position and the RELATIVE gain control fully clockwise to the switch detent position; set the FILTER selector in the 1 position.
- Remove power to extend circuit card A1 and re-apply power.
- With the sweep oscillator for 400 Hz, ± 4 Hz, adjust the Attenuator for 5 dB attenuation. Monitor the CHAN 1 SIGNAL INPUT with an rms voltmeter and adjust the sweep oscillator for 1 Vrms, ± 10 mV.
- On the SD1010B, adjust the FREQ control for maximum meter indication on the SD1012B. If the meter appears saturated (off scale), adjust the CHANNEL 1 FILTER 1-CAL control on the SD1012B for full scale.
- Check rms voltage at A1TP1; should be 31.6 mV (30.97 to 32.23 mV).
- At A1TP13, check for 0.234 Vrms (0.224 to 0.244 Vrms).
- Switch CHANNEL 1 RANGE (Vrms) switch to the .3 position and adjust the Attenuator for 3 dB attenuation (this increases the input level by 12 dB). Adjust A1R23 until the OVERLOAD lamp glows dimly.
- Adjust the Attenuator for 0 dB (increases level by 3 dB) and the OVERLOAD lamp should glow brightly.

Leave circuit card A1 extended and return RANGE switch to the 1 position.

4.2.4 Modulator (A2)

- a. Remove attenuator, and adjust oscillator to provide SIGNAL INPUT of 1 Vrms at 2 kHz.
- b. On AC Input/Output card A1, monitor A1TP3 with oscilloscope and adjust A1R32 for 8V p-p ($\pm 1V$ p-p).
- c. Remove power, remove card A1 and extender (do not replace the "key" at this time), and return A1 to its socket.

Re-apply power.

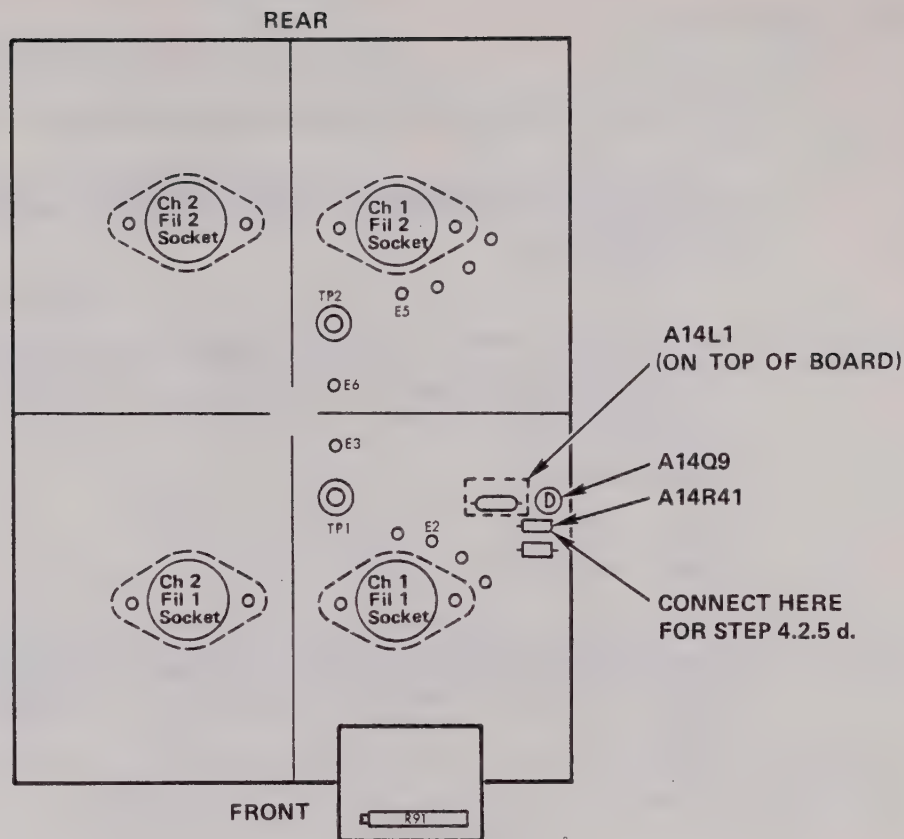
- d. Connect wave analyzer to A2TP2. Tune wave analyzer for 0 dB at 100 kHz.
- e. Alternately adjust A2R11 and A2C3 to decrease the 102-kHz signal to -50 dB or more below the 100-kHz signal. (Some fluxuations on the analyzer meter below -50 dB is normal.)

4.2.5 Filter Driver (A14)

CAUTION

To ensure stability of the crystal filters, be sure that at least 1 hour has elapsed since the power was initially applied to the instrument.

- a. Place instrument on right side (viewing instrument from the front) and remove the bottom cover to expose component side of card A14.
- b. Connect oscilloscope to J7, 100-kHz FIL I.F. On SD1010B, adjust front-panel FREQ control for maximum output at J7.
- c. Use oscilloscope and check that the gain is 5.6 or more between the following points: A14E2 (input) and A14TP1 (output), A14E5 (input) and A14TP2 (output). See drawing below for the relative location of these input and output points.



A14, Bottom View, Channel 1 Check Points

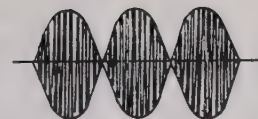
- d. Connect rms voltmeter to junction of A14Q9 base and A14R41. (See drawing above.)
- e. Sequentially set the FILTER switch to 1 and 2, and adjust the associated front-panel CAL control for approximately 360 mV at the base of A14Q9.
- f. Set FILTER switch to 1. Use non-metallic screwdriver and adjust A14L1 for maximum output at J7; recheck, on SD1010B, adjustment of front-panel FREQ control for maximum output.

Do not replace instrument bottom cover at this time.

4.2.6 AC Input/Output (A1), Output Section

- a. Remove power, extend card A1, and re-apply power.
- b. Use oscilloscope and check at A1TP3 for 8V p-p $\pm 1V$ p-p.

- c. Check at pin U of A1 card for 6V p-p (5.8V to 6.2V p-p).
- d. Check at A1TP6 for a single-sideband envelope, ≈ 360 mV p-p.
- e. Disconnect sweep oscillator from SIGNAL INPUT; connect a DVM to A1TP9.
- f. Adjust A1R71 for +15V (14V to 16V) at A1TP9.
- g. Reconnect sweep oscillator to SIGNAL INPUT and adjust for 400-Hz, 1 Vrms input.
- h. Connect rms voltmeter to A1TP9 and adjust A1R68 for 10 Vrms.
- i. Remove power, remove card A1 and extender, replace "key" between pins 13 and 14, and return A1 to its socket.



NOTE

If a 50-Hz BW filter is in position 2, set FILTER switch to 2 and adjust the FILTER 2 CAL adjust in the following procedure. Otherwise, select the FILTER that is available and adjust its associated CAL adjust.

- j. Insert a ratio transformer (RT) between the sweep oscillator output and the SIGNAL INPUT connector.
- k. Set RT to X0000 (0 dB) and adjust sweep oscillator for 400-Hz, 1 Vrms at the SIGNAL INPUT. Then set RT to 10000 (-20 dB).
- l. Connect rms voltmeter to J4, UNITY GAIN FIL SIG OUTPUT. Adjust applicable front-panel FILTER CAL adjust for 100 mV rms at J4.
- m. Set RT to X0000 (0 dB) and adjust A14R91 for 1 Vrms at J4. (R91 is a compensating potentiometer located on a small subassembly of card A14, accessible from the bottom of the instrument.)
- n. Repeat steps k. through m. as required for proper outputs at 0 dB and -20 dB input levels.
- o. Place instrument on bottom, but do not replace bottom cover at this time.

4.2.7 PSD (A4)

- a. Remove power, extend card A4 (save "key"), and re-apply power.
- b. Set RT to X0000 (0 dB) and ensure that the SIGNAL INPUT is 400-Hz, 1 Vrms.

- c. Check that the RANGE and METER MULTIPLIER switches are both set to 1, and set MODE switch to .01, SINE AVG (SEC).
- d. Adjust A4R13 for full-scale reading on front-panel Signal Amplitude meter.
- e. Set RT to 00300, connect DVM to A4TP6, and adjust A4R38 for 29.5 mV (29.3 to 29.7 mV). (Record reading for comparison in the next step.)
- f. Connect DVM to A4TP8 and adjust A4R54 for same reading as A4TP6 (± 0.1 mV).
- g. Set RT to X0000 (0 dB). Connect DVM to A4TP6 and adjust A4R44 for 10V (9.9V to 10.1V).
- h. Set RT to 00000. Set front-panel LOG DC ATTEN (dB) switch to -10.

Adjust A4R89 and A4R85 to mid-range of travel.

Connect jumper from A4TP8 to ground.
- i. Connect DVM to A4TP13 and adjust A4R69 for 0V ± 0.1 mV.

Remove jumper and return LOG DC ATTEN switch to (OPER) 0.
- j. Connect DVM to A4TP8 and adjust RT for 10V (9.9V to 10.1V) at A4TP8.
- k. Connect DVM to A4TP12 and adjust A4R89 for 400 mV (0 dB point).
- l. Connect DVM to A4TP8 and adjust RT for 100 mV at A4TP8.
- m. Connect DVM to A4TP12 and adjust A4R85 for 0V ± 1 mV (-40 dB point).
- n. Repeat steps j. through m. as necessary until the 0 dB and -40 dB points are correct.
- o. Connect DVM to A4TP8 and adjust RT for 10 mV at A4TP8 (approximately -60 dB).
- p. Recheck 0 dB and -40 dB points (steps j. through m.).
- q. Remove power, remove A4 and extender, replace the "key" between pins 17 and 18, and return A4 to its socket.
- r. If instrument is Model SD1012B-1, -2 (With Sine Reject), proceed to paragraph 4.2.8. If Model SD1012B, proceed to paragraph 4.2.9 and calibrate Channel 2.

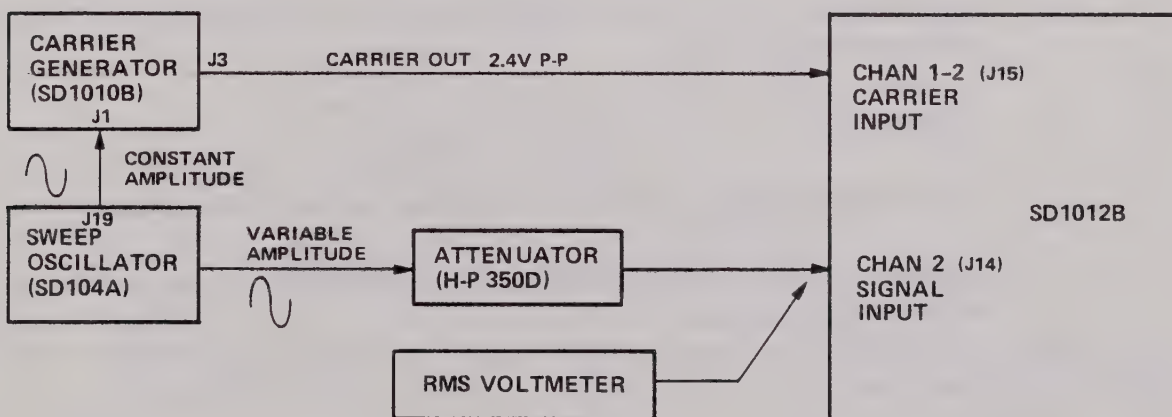
4.2.8 Sine Reject (A3) (Model SD1012B-1 only)

If available, a 5-Hz crystal filter is used for this adjustment; otherwise use lowest BW filter. Also, if two crystal filters were previously installed in Channel 1, remove the crystal filter from position No. 2. The following procedure assumes a 5-Hz BW filter in position No. 1, and the filter removed from position No. 2.

- a. Set FILTER switch to 1, RANGE switch to 1, and METER MULTIPLIER switch to 1. Adjust CHANNEL 1 SINE REJECT NULL to mid-range of travel.
- b. Apply 400-Hz, 1 Vrms (full-scale) signal to SIGNAL INPUT of SD1012B and TUNING INPUT of SD1010B. On SD1010B adjust front-panel TUNING INPUT AMPL adjust for center-line (green area) reading on TUNING AMPLITUDE meter.
- c. Calibrate FILTER 1 using the front-panel CONTROLS.
- d. Set FILTER switch to 2 (no filter installed).
- e. Connect rms voltmeter to J9, SINE REJECT OUTPUT, and adjust A3R26 for 1 Vrms ± 0.1 dB.
- f. Set FILTER switch to 1, and adjust front-panel Channel 1 ϕ control and A3R1 for minimum output at J9; should be at least -32 dB below 1 Vrms.
- g. Replace filter No. 2 (if applicable).

4.2.9 AC INPUT/OUTPUT (A8), Input Section (Channel 2)

Connect the SD1012B and test equipment as follows:



- a. On the SD1010B, set the OPER/CAL switch in the OPER position and adjust the TUNING AMPL control for a centerline indication (green area) on the TUNING AMPLITUDE meter.

- b. On Channel 2 of the SD1012B, set the RANGE (Vrms) and METER MULTIPLIER controls in the 1 position and the RELATIVE gain control fully clockwise to the switch detent position; set the FILTER selector in the 1 position.
 - c. Remove power to extend circuit card A8 and re-apply power.
 - d. With the sweep oscillator set for 400 Hz, ± 4 Hz, adjust the Attenuator for 5 dB attenuation. Monitor the CHAN 2 SIGNAL INPUT with an rms voltmeter and adjust the sweep oscillator for 1 Vrms, ± 10 mV.
 - e. On the SD1010B, adjust the FREQ control for maximum meter indication on the SD1012B. If the meter appears saturated (off scale), adjust the CHANNEL 2 FILTER 1-CAL control on the SD1012B for full scale.
 - f. Check rms voltage at A8TP1; should be 31.6 mV (30.97 to 32.23 mV).
 - g. At A8TP13, check for 0.234 Vrms (0.224 to 0.244 Vrms).
 - h. Switch CHANNEL 2 RANGE (Vrms) switch to the .3 position and adjust the Attenuator for 3 dB attenuation (this increases the input level by 12 dB). Adjust A8R23 until the OVERLOAD lamp glows dimly.
 - i. Adjust the Attenuator for 0 dB (increases level by 3 dB) and the OVERLOAD lamp should glow brightly.
- Leave circuit card A8 extended and return RANGE switch to the 1 position.

4.2.10 Modulator (A6)

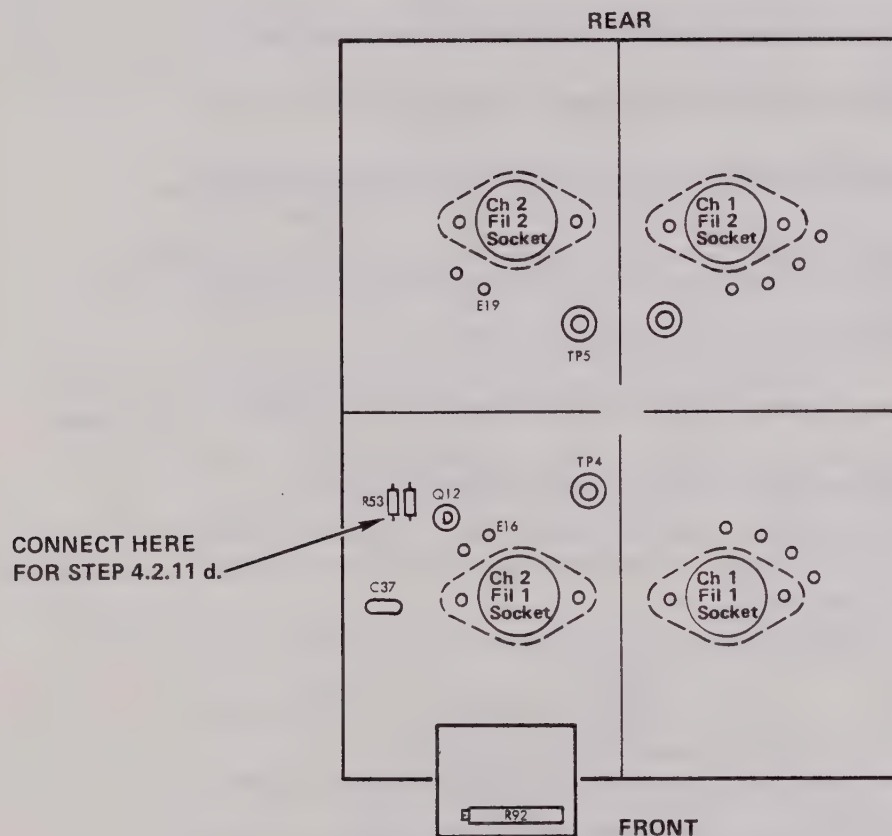
- a. Remove attenuator, and adjust oscillator to provide SIGNAL INPUT of 1 Vrms at 2 kHz.
 - b. On AC Input/Output card A1, monitor A1TP3 with oscilloscope and adjust A1R32 for 8V p-p.
 - c. Remove power, remove card A1 and extender (do not replace the "key" at this time), and return A1 to its socket.
- Re-apply power.
- d. Connect wave analyzer to A6TP2. Tune wave analyzer for 0 dB at 100 kHz.
 - e. Alternately adjust A6R11 and A6C3 to decrease the 102-kHz signal to -50 dB or more below the 100-kHz signal.

4.2.11 Filter Driver (A14)

CAUTION

To ensure stability of the crystal filters, be sure that at least 1 hour has elapsed since the power was initially applied to the instrument.

- a. Place instrument on right side (viewing instrument from the front).
- b. Connect oscilloscope to J19, 100-kHz FIL I.F. On SD1010B, adjust front-panel FREQ control for maximum output.
- c. Use oscilloscope and check that the gain is 5.6 or more between the following points: A14E16 (input) and A14TP4 (output), A14E19 (input) and A14TP5 (output). See drawing below for the relative location of these input and output points.



A14, Bottom View, Channel 2 Check Points

- d. Connect rms voltmeter to base of A14Q12, junction of A14R53 and A14C37. (See drawing above.)
- e. Sequentially set the FILTER switch to 1 and 2, and adjust the associated front-panel CAL control for approximately 360 mV at base of A14Q12.

- f. Set FILTER switch to 1. Use non-metallic screwdriver and adjust A14L4 for maximum output at J19; recheck, on SD1010B, adjustment of front-panel FREQ control for maximum output.
- g. Replace cover over crystal filters. (Do not replace instrument bottom cover at this time.)

4.2.12 AC Input/Output (A8), Output Section

- a. Remove power, extend card A8, and re-apply power.
- b. Use oscilloscope and check at A8TP3 for 8V p-p (7V to 9V p-p).
- c. Check at pin U of A8 card for 6V p-p (5.8V to 6.2V p-p).
- d. Check at A8TP6 for a single-sideband envelope, approximately 360 mV p-p.
- e. Disconnect sweep oscillator from SIGNAL INPUT; connect a DVM to A8TP9.
- f. Adjust A8R71 for +15V (+14V to +16V) at A8TP9.
- g. Reconnect sweep oscillator to SIGNAL INPUT and adjust for 400-Hz, 1 Vrms input.
- h. Connect rms voltmeter to A8TP9 and adjust A8R68 for 10 Vrms.
- i. Remove power, remove card A8 and extender, replace "key" between pins 13 and 14, and return A8 to its socket.

NOTE

If a 50-Hz BW filter is in position 2, set FILTER switch to 2 and adjust the FILTER 2 CAL adjust in the following procedure. Otherwise, select the FILTER that is available and adjust its associated CAL adjust.

- j. Insert a ratio transformer (RT) between the sweep oscillator output and the SIGNAL INPUT connector.
- k. Set RT to X0000 (0 dB) and adjust sweep oscillator for 400-Hz, 1 Vrms at the SIGNAL INPUT. Then set RT to 10000 (-20 dB).
- l. Connect rms voltmeter to J16, UNITY GAIN FIL SIG OUTPUT. Adjust applicable front-panel FILTER CAL adjust for 100 mV rms at J16.

- m. Set RT to X0000 (0 dB) and adjust A14R92 for 1 Vrms at J12. (R92 is a compensating potentiometer located on a small subassembly of card A14, accessible from the bottom of the instrument.)
- n. Repeat steps k. through m. as required for proper outputs at 0 dB and -20 dB input levels.
- o. Replace bottom cover and place instrument on bottom.

4.2.13 PSD (A5)

- a. Remove power, extend card A5 (save "key"), and re-apply power.
- b. Set RT to X0000 (0 dB) and ensure that the SIGNAL INPUT is 400-Hz, 1 Vrms.
- c. Check that the RANGE and METER MULTIPLIER switches are both set to 1, and set MODE switch to .01, SINE AVG (SEC).
- d. Adjust A5R13 for full-scale reading on front-panel Signal Amplitude meter.
- e. Set RT to 00300, connect DVM to A5TP6, and adjust A5R38 for 29.5 mV (29.3 to 29.7 mV). (Record reading for comparison in the next step.)
- f. Connect DVM to A5TP8 and adjust A5R54 for same reading as A5TP6 (± 0.1 mV).
- g. Set RT to X0000 (0 dB). Connect DVM to A5TP6 and adjust A5R44 for 10V (9.9 to 10.1V).
- h. Set RT to 00000. Set front-panel LOG DC ATTEN (dB) switch to -10.

Adjust A5R89 and A5R85 to mid-range of travel.

Connect jumper from A5TP8 to ground.
- i. Connect DVM to A5TP13 and adjust A5R69 for $0V \pm 0.1$ mV.

Remove jumper and return LOG DC ATTEN switch to (OPER) 0.
- j. Connect DVM to A5TP8 and adjust RT for 10V at A5TP8.
- k. Connect DVM to A5TP12 and adjust A5R89 for 400 mV (0 dB point).
- l. Connect DVM to A5TP8 and adjust RT for 100 mV at A5TP8.
- m. Connect DVM to A5TP12 and adjust A5R85 for $0V \pm 1$ mV (-40 dB point).

- n. Repeat steps j. through m. as necessary until the 0 dB and -40 dB points are correct.
- o. Connect DVM to A5TP8 and adjust RT for 10 mV at A5TP8 (approximately -60 dB).
- p. Recheck 0 dB and -40 dB points (steps j. through m.).
- q. Remove power, remove A5 and extender, replace the "key" between pins 17 and 18, and return A5 to its socket.
- r. If instrument is Model SD1012B-2 (With Sine Reject in Channels 1 and 2), proceed to paragraph 4.2.14. If Model SD1012B(-1), disconnect all test equipment, replace instrument top cover, and place instrument in service. To ensure overall operation, perform the operational checkout procedure of paragraph 2.2 in Section II in this manual.

4.2.14 Sine Reject (A7) (Model SD1012B-2 only)

If available, a 5-Hz crystal filter is used for this adjustment; otherwise use lowest BW filter. Also, if two crystal filters were previously installed in Channel 2, remove the crystal filter from position No. 2. The following procedure assumes a 5-Hz BW filter in position No. 1, and the filter removed from position No. 2.

- a. Set FILTER switch to 1, RANGE switch to 1, and METER MULTIPLIER switch to 1. Adjust CHANNEL 2 SINE REJECT NULL to mid-range of travel.
- b. Apply 400-Hz, 1 Vrms (full-scale) signal to SIGNAL INPUT of SD1012B and TUNING INPUT of SD1010B. On SD1010B, adjust front-panel TUNING INPUT AMPL adjust for center-line (green area) reading on TUNING AMPLITUDE meter.
- c. Calibrate FILTER 1 using the front-panel controls.
- d. Set FILTER switch to 2 (no filter installed).
- e. Connect rms voltmeter to J21, SINE REJECT OUTPUT, and adjust A7R26 for 1 Vrms.
- f. Set FILTER switch to 1, and adjust front-panel Channel 2 ϕ control and A7R1 for minimum output at J21; should be at least -32 dB below 1 Vrms.

Disconnect power cord from primary power source, replace filter No. 2, replace filter compartment cover and instrument top cover, and place instrument in service. To ensure overall operation, perform the operational checkout procedure of paragraph 2.2 in Section II of this manual.

4.3 PARTS REPLACEMENT

Except for some special SDC specification-controlled parts listed in Table 4-2, which must be ordered from Spectral Dynamics Corporation, standard electronic parts have been used throughout the instrument. Spectral Dynamics Corporation maintains a complete stock of all parts and complete printed circuit card assemblies used in the instrument.

When ordering a replacement part or assembly, be sure to include the reference designation, the part number or manufacturer's designation, and a complete description; include also the instrument name, model number, and serial number.

Table 4-2. Special Replaceable Parts List

Reference Designator	Description	Part Number Mfr. Designation
A1, A8	AC Input/Output Assembly	10942-1
A2, A6	Modulator Assembly	11204-1
A3, A7	Sine Reject Assembly	10954-1
A4, A5	PSD Output Assembly	10948-1
A9	Power Supply Assembly	10958-1
A10, A11, A12, A13	PSD Plug-In Assembly	11312-*
A14	Filter Driver Assembly	11188-1
-----	Printed-Circuit Board Extender	23060-1
A15	Filter Phase Assembly	11210-1

(Cont'd)

*See DWG. NO. 11386, Sheet 2 of 4, for Assembly Dash Number Details.

Table 4-2. Special Replaceable Parts List
(Cont'd)

Reference Designator	Description	Part Number Mfr. Designation
A4A1, A5A1	709C (Factory Selected)	20013-244
DS1, DS4	OVERLOAD Indicator Lamps	CF03-ACS-2102
DS2, DS3	FILTER Indicator Lamps	CF03-WTS-2102
M1, M2	Meter, Signal Amplitude	20013-198
P11	AUX FUNCTIONS, Jumpered Plug	SDC 11514
P22	AUX FUNCTIONS, Jumpered Plug	SDC 11515
S1/S4/R18, S7/S9/R19	RANGE (Vrms) Switch/CAL Switch/ RELATIVE Control	20013-165
S2/S6, S8/S10	METER MULTIPLIER/LOG DC ATTEN (dB)	20013-164
S3	FILTER Switch	20013-207
S5	MODE Switch	20013-209
S11/DS5	POWER Switch/Indicator	10529000
T1	Transformer, Power	20013-195
	Power Cord	Belden 17258-S

APPENDIX A

CALCULATING VALUES OF SWEEP RATE CONTROL & AVERAGING TIME RESISTORS

USED IN PSD PLUG-IN ASSEMBLIES (11312)

The standard PSD Plug-in Assemblies (11312) for the SD1012B are designed for use over the frequency range of 5 Hz to 5 kHz; for example, a Model SD104A-5 Sweep Oscillator with its MULTIPLIER (A) switch set to 1, or a Model SD114 Sweep Oscillator Servo. (Refer to Table A-1.) In these cases, the oscillators have a maximum sweep rate of 100 Hz/s and a sweep rate sensitivity of 0.1 V/Hz/s. (Sweep rate sensitivity equals 10 volts divided by the maximum sweep rate.) For proper sweep rate control when operating in other than the 5 Hz to 5 kHz frequency range, the particular resistor (R5, R6, R7, or R8) on assembly 11312 that is associated with the desired PSD CONFIDENCE (MODE switch) setting, must be replaced. The correct resistor value is calculated as follows:

- a. Determine the filter bandwidth (BW) to be used and the "number of degrees of freedom" (N) desired. Refer to Table 2-3 in the basic SD1012B manual and determine which PSD CONFIDENCE switch position should be used. The associated resistors on assembly 11312 for each switch position are as follows:

<u>PSD Confidence Switch Position</u>	<u>Resistor on Assembly 11312</u>
1	R4 & R5
2	R3 & R6
3	R2 & R7
4	R1 & R8

- b. Compute the analysis sweep rate (SR):

$$\text{Analysis SR (Hz/s)} = \frac{\text{BW}^2}{N}$$

(The analysis sweep rate for all SDC standard filter bandwidths, 1.5 Hz through 200 Hz, is given in Table 2-3 of the SD1012B manual.)

- c. Using the computed analysis sweep rate, SR, calculate the sweep rate control voltage (V_C) to be supplied by the PSD Plug-in Assembly:

$$V_C = \frac{\text{SR} \times 10\text{V}}{\text{SR}_{\text{max}}}$$

Since

$$\frac{10\text{V}}{\text{SR}_{\text{max}}} = \text{SR sensitivity,}$$

$$V_C = \text{SR} \times \text{SR sensitivity}$$

TABLE A-1

MODEL NUMBER	FREQUENCY RANGE	MULTIPLIER(A) SETTING	SR Max (Hz/s) w/10 Vdc Input	SR Sensitivity (V/Hz/s)
SD104A-5	50 Hz - 50 kHz	10	1000	0.01
	* 5 Hz - 5 kHz	* 1	100	0.1
	0.5 Hz - 500 Hz	.1	10	1
	0.05 Hz - 50 Hz	.01	1	10
	0.005 Hz - 5 Hz	.001	0.1	100
SD104A-2	20 Hz - 20 kHz	10	1000	0.01
	* 2 Hz - 2 kHz	* 1	100	0.1
	0.2 Hz - 200 Hz	.1	10	1
	0.02 Hz - 20 Hz	.01	1	10
SD104A-1	10 Hz - 10 kHz	10	200	0.05
	1 Hz - 1 kHz	1	20	0.5
	0.1 Hz - 100 Hz	.1	2	5
	0.01 Hz - 10 Hz	.01	0.2	50
SD104-10	20 Hz - 20 kHz	20	400	0.025
	10 Hz - 10 kHz	10	200	0.05
	* 5 Hz - 5 kHz	* 5	100	0.1
	2 Hz - 2 kHz	2	40	0.25
	1 Hz - 1 kHz	1	20	0.5
	0.5 Hz - 500 Hz	.5	10	1
	0.2 Hz - 200 Hz	.2	4	2.5
	0.1 Hz - 100 Hz	.1	2	5
	0.05 Hz - 50 Hz	.05	1	10
	0.02 Hz - 20 Hz	.02	0.4	25
SD114	* 5 Hz - 5 kHz	*-----	100	0.1

*No changes are required to standard PSD Plug-In Assemblies 11312 when operating in this range.

- d. Determine value of associated sweep rate resistor (R5, R6, R7 or R8):

$$R = \frac{V_C \times 15,000}{15 \text{ volts} - V_C}$$

In the above equation, 15,000 is the approximate total resistance of the series divider network (assuming that the Sweep Rate Adjust, 2k ohm potentiometer is centered); and 15 volts is the supply voltage across the resistor divider network.

- e. Determine value of associated averaging time resistor (R1, R2, R3 or R4):

$$R = \frac{N}{4 \text{ BW}} \times 100 \text{ k}\Omega$$

The following is a typical example:

Given:

Frequency range of operation is 50 Hz to 50 kHz (Source is an SD104A-5 Sweep Oscillator with its MULTIPLIER (A) switch set to 10.)

FILTER BW = 10 Hz; desired N = 200.

From Table 2-3 in SD1012B manual, with BW = 10 and N = 200, PSD CONFIDENCE switch position 4 is selected. Therefore, associated resistor R8 must be calculated and replaced with the closest value of 1% 1/4W resistor.

Proceed as follows:

- 1) Calculate required analysis sweep rate, SR:

$$SR = \frac{BW^2}{N} = \frac{10^2}{200} = 0.5 \text{ Hz/s}$$

- 2) Calculate sweep rate control voltage, V_C :

From Table A-1, the SR sensitivity is 0.01 for MULTIPLIER (A) switch setting 10 on the SD104A-5. Therefore,

$$V_C = SR \times SR \text{ sens} = 0.5 \text{ Hz/s} \times 0.01 \text{ V/Hz/s} = 0.005 \text{ Volts}$$

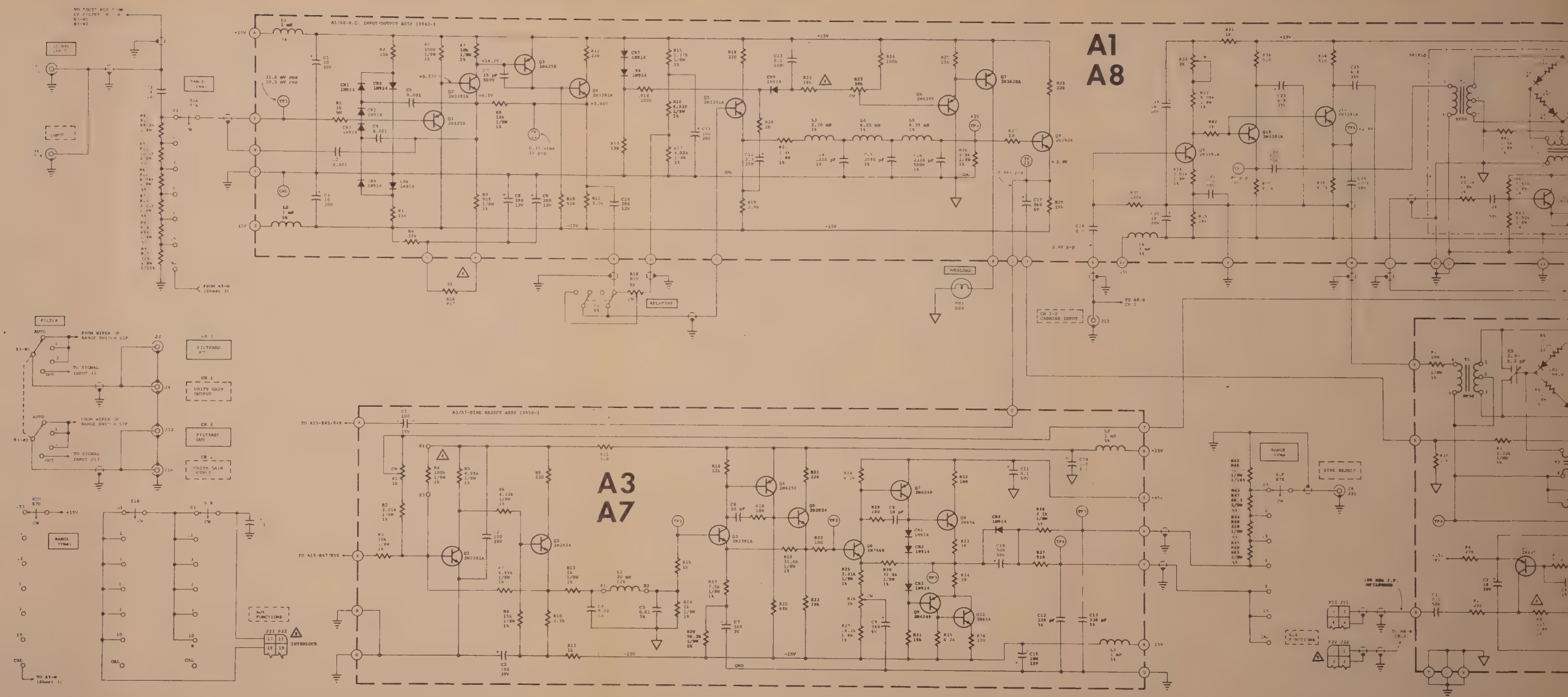
- 3) Calculate R8:

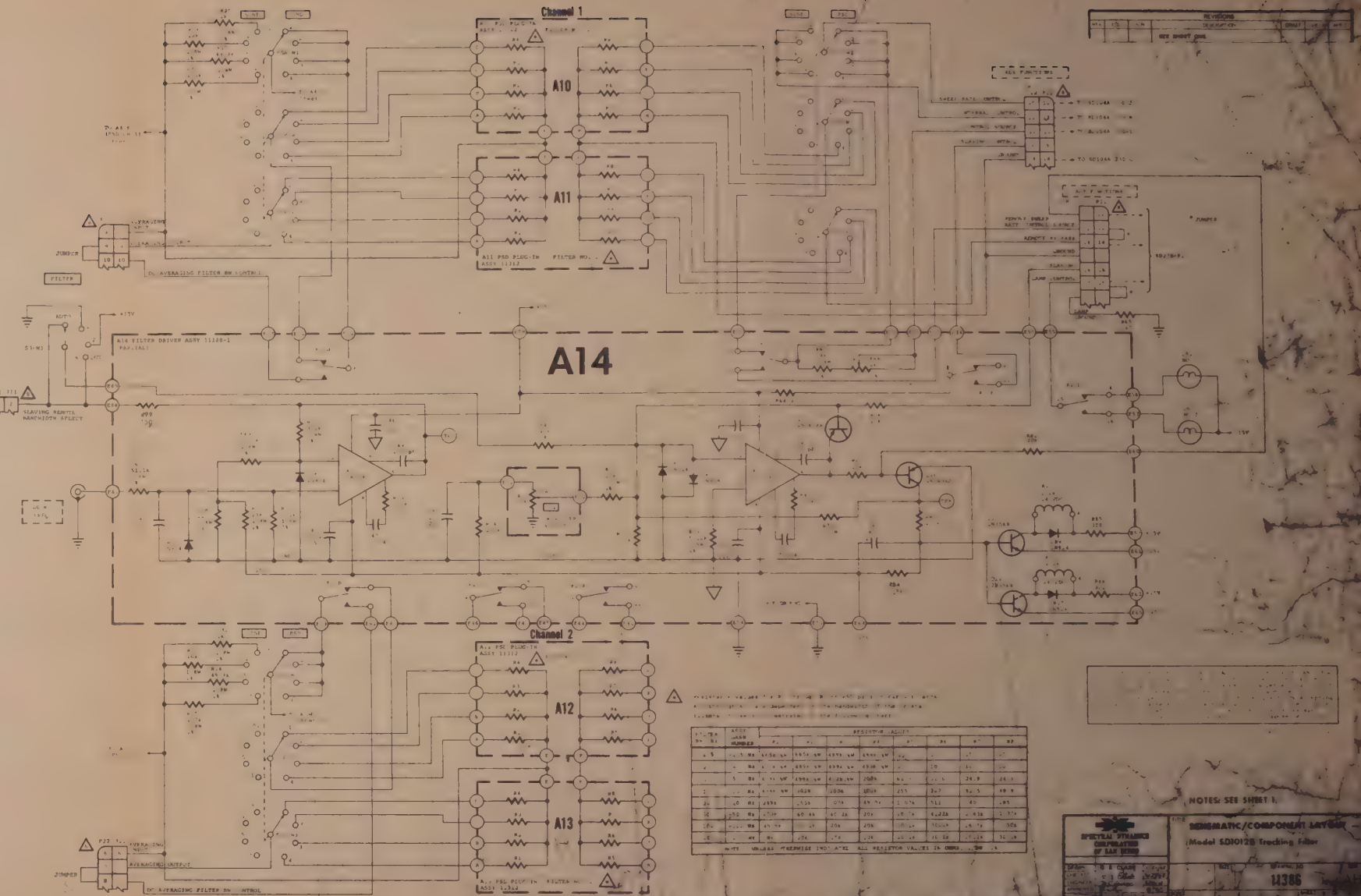
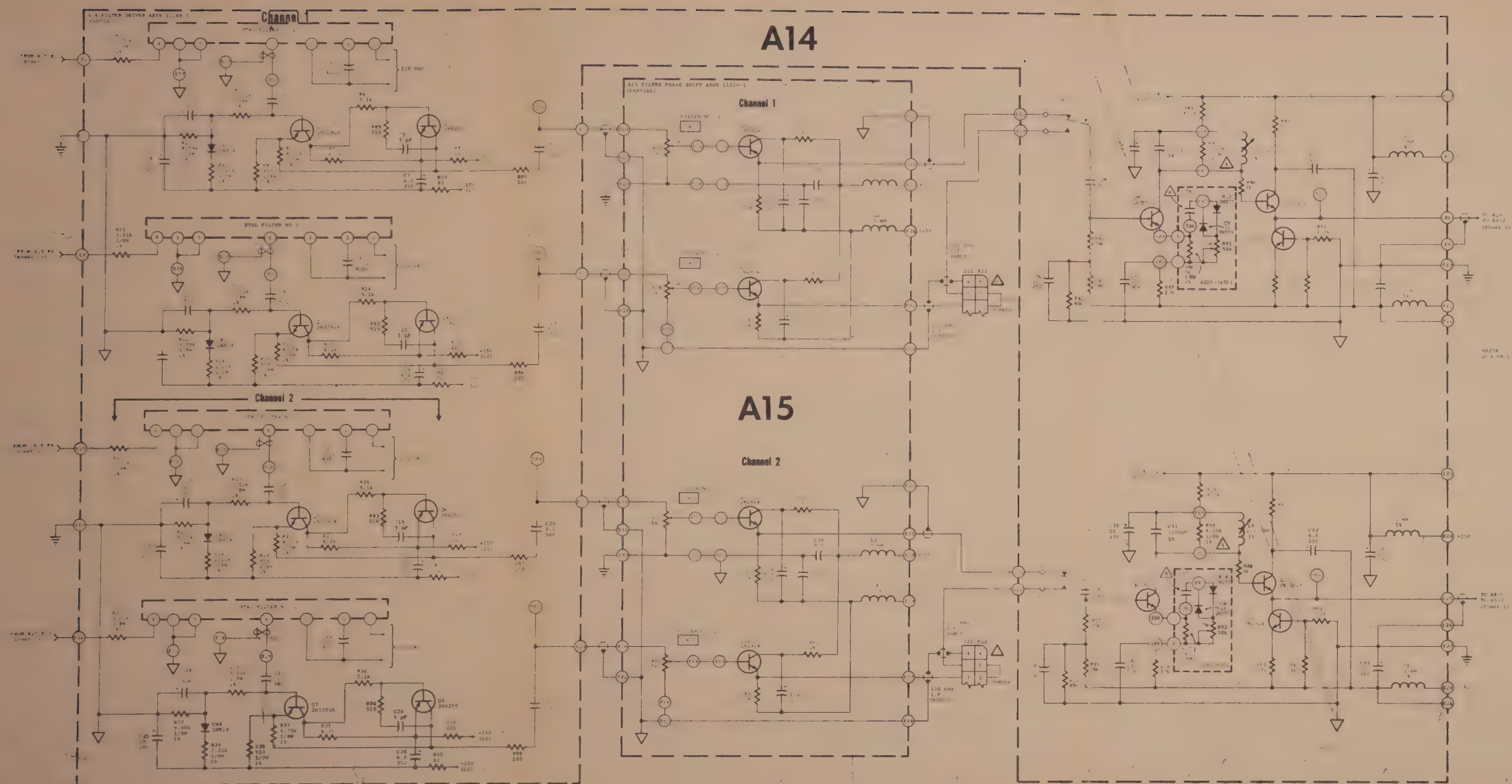
$$R8 = \frac{V_C \times 15,000}{15 \text{ volts} - V_C} = \frac{0.005 \times 15,000}{15 - 0.005} = \approx 5 \text{ ohms}$$

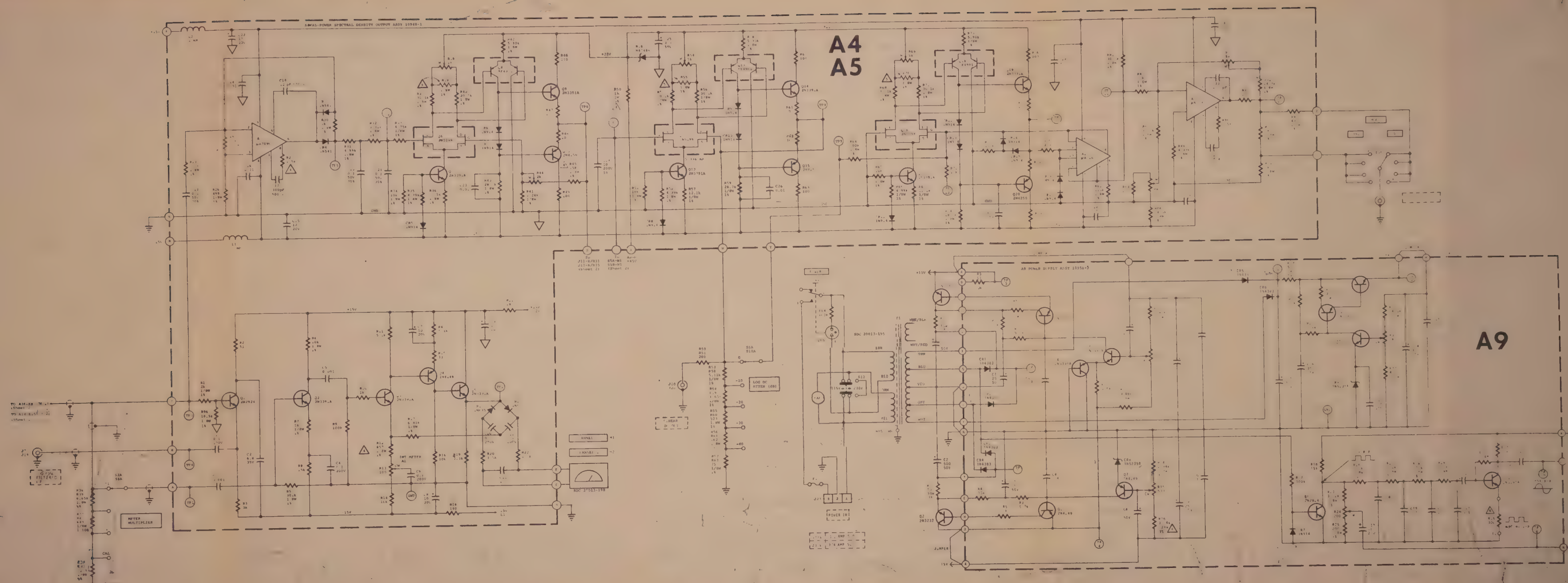
Replace R8 with a 4.99 ohm 1% 1/4W resistor.

- 4) Calculate R1: $R1 = \frac{N}{4 \text{ BW}} \times 100k = 500k$

Replace R1 with a 499k 1% 1/4W resistor.



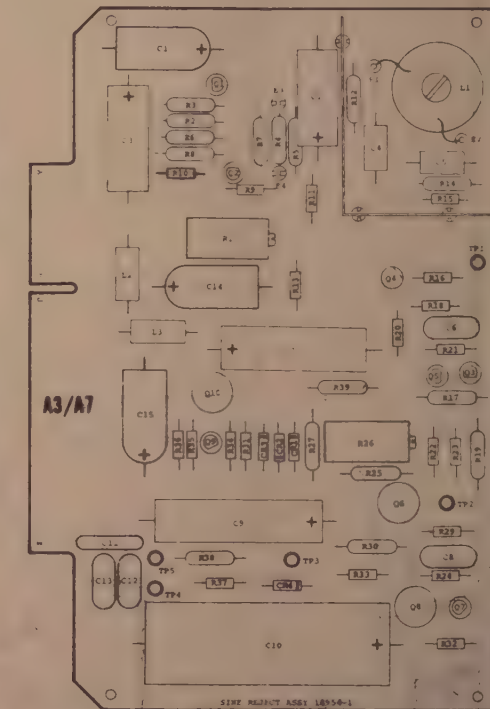
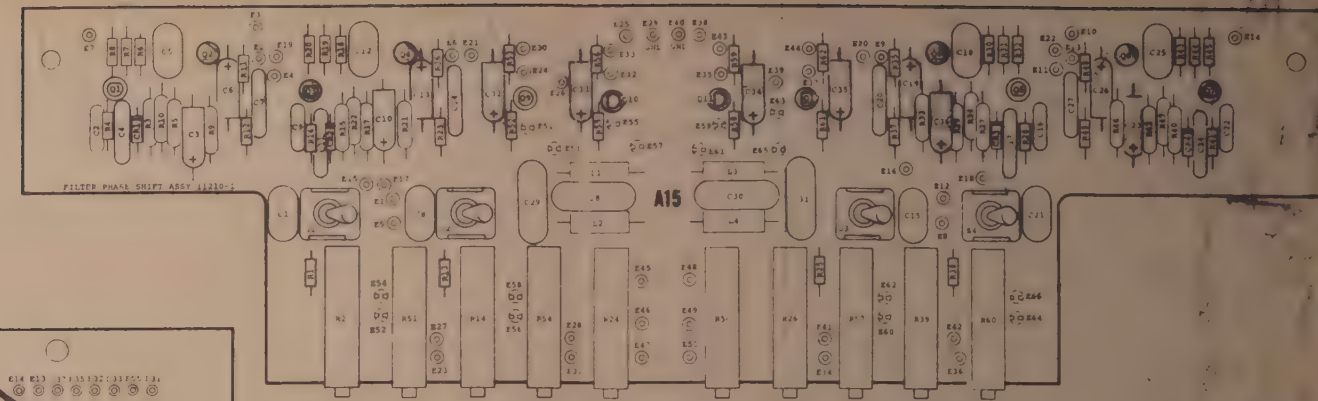
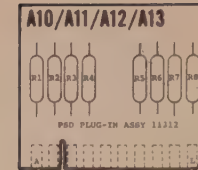
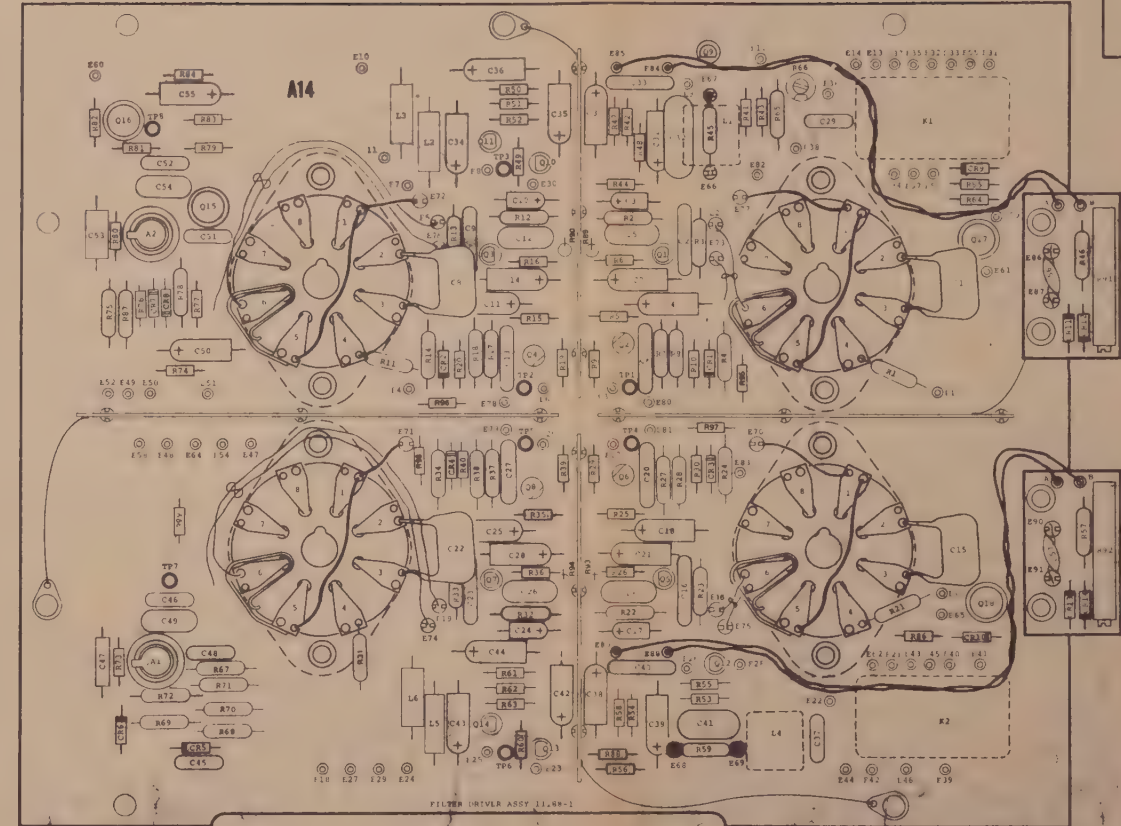
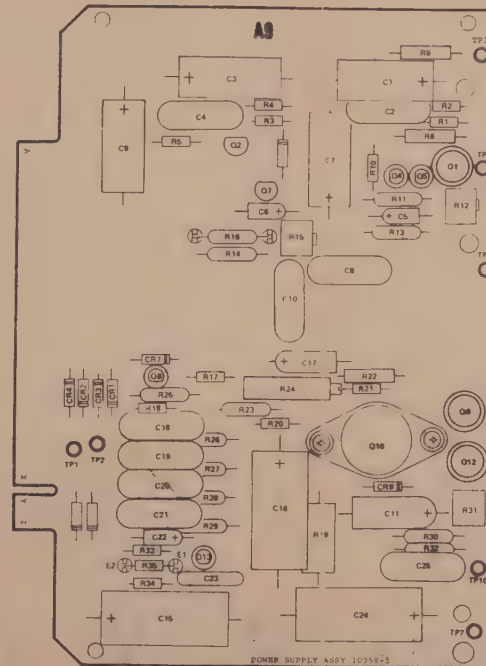
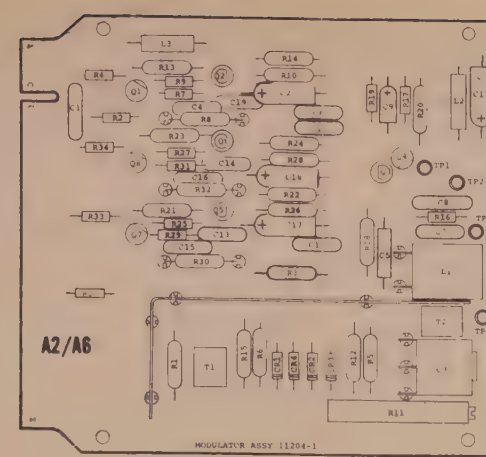
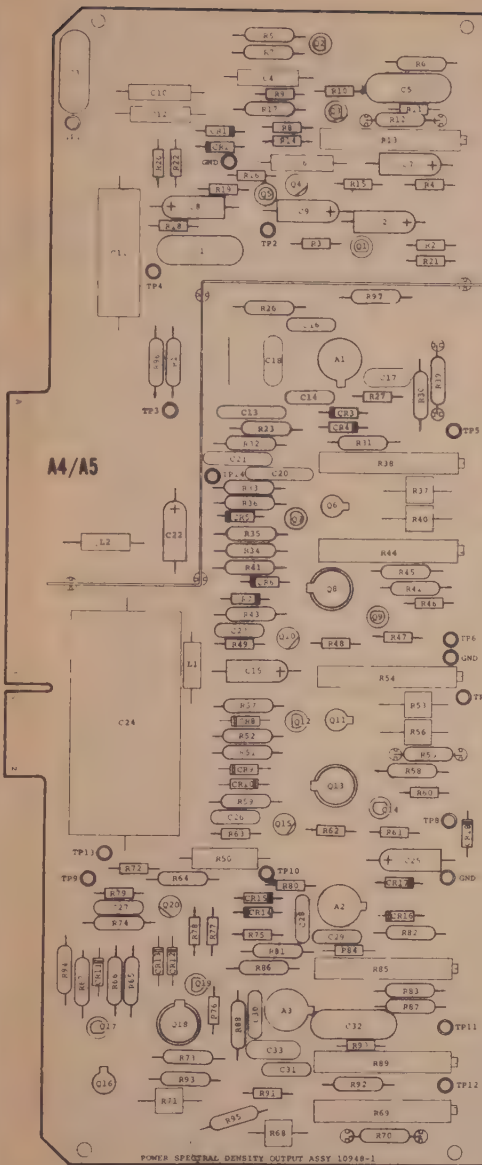
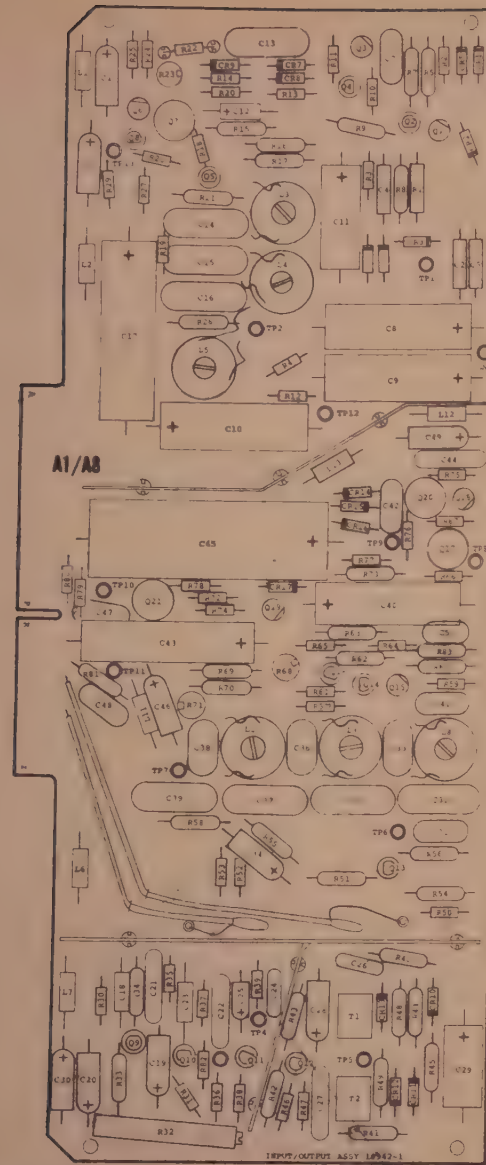




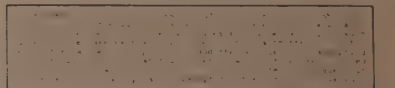
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NOTE: SEE SHEET 1

SPECTRAL DYNAMICS CORPORATION OF SAN JOSE		SCHEMATIC COMPONENT LAYOUT Model SD1012 Tracking Filter	
C. E. CLARK V. J. CLARK J. E. CLARK J. E. CLARK		11386 AH	



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NOTES: SEE SHEET 1.	
SPECTRAL DENSITY CALCULATION OF SINE WAVE MODEL SD1012B Tracking Filter DATE: 11/18/66 BY: JAH CHKD: JAH APP'D: JAH	SCHEMATIC COMPONENT LAYOUT Model SD1012B Tracking Filter DATE: 11/18/66 BY: JAH CHKD: JAH APP'D: JAH
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APPENDIX A

CALCULATING VALUES OF SWEEP RATE CONTROL & AVERAGING TIME RESISTORS USED IN PSD PLUG-IN ASSEMBLIES (11312)

The standard PSD Plug-in Assemblies (11312) for the SD1012B are designed for use over the frequency range of 5 Hz to 5 kHz; for example, a Model SD104A-5 Sweep Oscillator with its MULTIPLIER (A) switch set to 1, or a Model SD114 Sweep Oscillator Servo. (Refer to Table A-1.) In these cases, the oscillators have a maximum sweep rate of 100 Hz/s and a sweep rate sensitivity of 0.1 V/Hz/s. (Sweep rate sensitivity equals 10 volts divided by the maximum sweep rate.) For proper sweep rate control when operating in other than the 5 Hz to 5 kHz frequency range, the particular resistor (R5, R6, R7, or R8) on assembly 11312 that is associated with the desired PSD CONFIDENCE (MODE switch) setting, must be replaced. The correct resistor value is calculated as follows:

- a. Determine the filter bandwidth (BW) to be used and the "number of degrees of freedom" (N) desired. Refer to Table 2-3 in the basic SD1012B manual and determine which PSD CONFIDENCE switch position should be used. The associated resistors on assembly 11312 for each switch position are as follows:

<u>PSD Confidence Switch Position</u>	<u>Resistor on Assembly 11312</u>
1	R4 & R5
2	R3 & R6
3	R2 & R7
4	R1 & R8

- b. Compute the analysis sweep rate (SR):

$$\text{Analysis SR (Hz/s)} = \frac{\text{BW}^2}{N}$$

(The analysis sweep rate for all SDC standard filter bandwidths, 1.5 Hz through 200 Hz, is given in Table 2-3 of the SD1012B manual.)

- c. Using the computed analysis sweep rate, SR, calculate the sweep rate control voltage (V_C) to be supplied by the PSD Plug-in Assembly:

$$V_C = \frac{\text{SR} \times 10\text{V}}{\text{SR}_{\text{max}}}$$

Since

$$\frac{10\text{V}}{\text{SR}_{\text{max}}} = \text{SR sensitivity,}$$

$$V_C = \text{SR} \times \text{SR sensitivity}$$

TABLE A-1

MODEL NUMBER	FREQUENCY RANGE	MULTIPLIER(A) SETTING	SR Max (Hz/s) w/10 Vdc Input	SR Sensitivity (V/Hz/s)
SD104A-5	50 Hz - 50 kHz	10	1000	0.01
	* 5 Hz - 5 kHz	* 1	100	0.1
	0.5 Hz - 500 Hz	.1	10	1
	0.05 Hz - 50 Hz	.01	1	10
	0.005 Hz - 5 Hz	.001	0.1	100
SD104A-2	20 Hz - 20 kHz	10	1000	0.01
	* 2 Hz - 2 kHz	* 1	100	0.1
	0.2 Hz - 200 Hz	.1	10	1
	0.02 Hz - 20 Hz	.01	1	10
SD104A-1	10 Hz - 10 kHz	10	200	0.05
	1 Hz - 1 kHz	1	20	0.5
	0.1 Hz - 100 Hz	.1	2	5
	0.01 Hz - 10 Hz	.01	0.2	50
SD104-10	20 Hz - 20 kHz	20	400	0.025
	10 Hz - 10 kHz	10	200	0.05
	* 5 Hz - 5 kHz	* 5	100	0.1
	2 Hz - 2 kHz	2	40	0.25
	1 Hz - 1 kHz	1	20	0.5
	0.5 Hz - 500 Hz	.5	10	1
	0.2 Hz - 200 Hz	.2	4	2.5
	0.1 Hz - 100 Hz	.1	2	5
	0.05 Hz - 50 Hz	.05	1	10
	0.02 Hz - 20 Hz	.02	0.4	25
SD114	* 5 Hz - 5 kHz	*-----	100	0.1

*No changes are required to standard PSD Plug-In Assemblies 11312 when operating in this range.

- d. Determine value of associated sweep rate resistor (R5, R6, R7 or R8):

$$R = \frac{V_C \times 15,000}{15 \text{ volts} - V_C}$$

In the above equation, 15,000 is the approximate total resistance of the series divider network (assuming that the Sweep Rate Adjust, 2k ohm potentiometer is centered); and 15 volts is the supply voltage across the resistor divider network.

- e. Determine value of associated averaging time resistor (R1, R2, R3 or R4):

$$R = \frac{N}{4 \text{ BW}} \times 100 \text{ k}\Omega$$

The following is a typical example:

Given:

Frequency range of operation is 50 Hz to 50 kHz (Source is an SD104A-5 Sweep Oscillator with its MULTIPLIER (A) switch set to 10.)

FILTER BW = 10 Hz; desired N = 200.

From Table 2-3 in SD1012B manual, with BW = 10 and N = 200, PSD CONFIDENCE switch position 4 is selected. Therefore, associated resistor R8 must be calculated and replaced with the closest value of 1% 1/4W resistor.

Proceed as follows:

- 1) Calculate required analysis sweep rate, SR:

$$SR = \frac{BW^2}{N} = \frac{10^2}{200} = 0.5 \text{ Hz/s}$$

- 2) Calculate sweep rate control voltage, V_C :

From Table A-1, the SR sensitivity is 0.01 for MULTIPLIER (A) switch setting 10 on the SD104A-5. Therefore,

$$V_C = SR \times SR \text{ sens} = 0.5 \text{ Hz/s} \times 0.01 \text{ V/Hz/s} = 0.005 \text{ Volts}$$

- 3) Calculate R8:

$$R8 = \frac{V_C \times 15,000}{15 \text{ volts} - V_C} = \frac{0.005 \times 15,000}{15 - 0.005} = \approx 5 \text{ ohms}$$

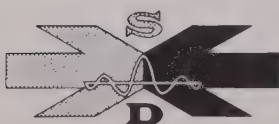
Replace R8 with a 4.99 ohm 1% 1/4W resistor.

- 4) Calculate R1: $R1 = \frac{N}{4 \text{ BW}} \times 100\text{k} = 500\text{k}$

Replace R1 with a 499k 1% 1/4W resistor.

SOME ANALOG METHODS OF POWER SPECTRAL DENSITY ANALYSIS

**By
L. R. Burrow, Jr.**



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SOME ANALOG METHODS FOR POWER SPECTRAL DENSITY ANALYSIS

By: Laurie R. Burrow, Jr., Spectral Dynamics Corporation of San Diego

INTRODUCTION

The analysis of stationary random signals has been the subject of many papers, articles, and books.

The basic ingredients of a system are well known; however, certain advances in available analysis equipment prompts a review of the methods with an eye toward reducing the required analysis time, while providing adequate resolution and statistical accuracy. Minimizing the required analysis time reduces the cost of data reduction in terms of dollars and calendar time.

WHAT WE ARE ANALYZING

The signals occurring in nature can generally be separated into seven general categories; however, there are many possible combinations. Since it is possible for the signals to be analyzed to be made up of various combinations, a review of the basic types of signals follows:

Stationary Periodic Signals

Examples:

$$Y(t) = K \sin(\omega t) \quad (1)$$

Where: K and ω = constants.

This says that the function repeats itself exactly for all values of t . A function $Y(t)$ can be made up of harmonically related sinusoids and can be expanded into a Fourier series with each harmonic component also repeating itself exactly for all values of t :

$$Y(t) = K_1 (\sin 2\pi f_1 t + \theta_1) + K_2 (\sin 2\pi f_2 t + \theta_2) + \dots \quad (2)$$

or

$$Y(t) = \sum_{n=0}^{n=\infty} K_n (\sin 2\pi n f_1 t + \theta_n) \quad (3)$$

Where:

K_n = the peak amplitude of the n th harmonic

f_1 = the fundamental frequency

θ_n = the phase of the n th harmonic with respect to the fundamental

Stationary Complex Signals

Generally, this may be thought of as a mixture of sinusoids where the various signals in the mixture are not necessarily harmonically related and the amplitudes of the individual components are not changing.

The mixture may be produced by the summing of sinusoids and the multiplication of sinusoids or any combination.

Example:

$$Y(t) = K_1 \sin \omega_1 t + K_2 \cos \omega_2 t + K_3 (\cos \omega_3 t) (\cos \omega_4 t) \text{ etc.} \quad (4)$$

Quasi-Sinusoid Signals

(Sometimes referred to as a Phase-Coherent Function)

This is a sinusoidal function whose amplitude and frequency vary slowly and smoothly with respect to time and is reproducible under the same conditions that originally produced it.

$$Y(t) = Y_1(t) \sin [\omega(t) + \theta] \quad (5)$$

Where:

$Y_1(t)$ and $\omega(t)$ = functions of time

θ = phase angle with respect to the time axis

Stationary Random Signals

In the cases discussed above, the functions can be determined from their magnitude vs. time histories and the functions can generally be made to repeat. With random signals, the functions cannot be determined explicitly and have to be treated in a probabilistic manner.

Thus, statistics and statistical averages are used to define the characteristics of random signals.

A given stationary random signal is described in the amplitude domain by its probability density function and its spectral characteristics by the power spectral density function.

The auto-correlation function is another function that describes the random signal in the time domain, but will not be discussed in this paper.

Transient Signals

These signals are time varying functions, occurring over a restricted time interval.

Non-Stationary Periodic or Complex Signals

$$Y(t) = Y(t, f, \text{etc.}) \sin [\omega(t, f, \text{etc.}) + \theta] \quad (6)$$

Where:

$Y(t, f, \text{etc.})$ = a function of an independent variable

$\omega(t, f, \text{etc.})$ = a function of an independent variable

This could be expanded to the Fourier series for harmonic related signals and expanded for a complex signal. (Equations 3 and 4)

Non-Stationary Random Signals

These signals are very difficult to analyze in that the average amplitude characteristics change with time as can its spectral characteristics.

Random signals to be analyzed can be generally classed as from two sources:

- (1) Actual signals from a device experiencing the "in use" conditions. (Such as test firings, flight tests, etc.)
- (2) Signals from a device experiencing simulated "in use" conditions. (Such as from a vibration test machine, etc.)

The conditions of (2) above are generally easier to control and analyze.

The signal from (1) above is the most difficult to handle since several factors may be present that affect the analysis.

These considerations are:

- (1) Is the signal truly random or does it contain periodic components.
- (2) Is the signal stationary.
- (3) Does the signal contain transients.

It is important to separate or at least identify the sinusoidal signals from the random signals so that proper interpretation of an analysis record can be made.

If shocks are present, they should be separated from the record and analyzed by appropriate methods.

Stationarity of the data is quite often assumed under certain conditions and the analysis carried out as if the data were stationary. A more detailed discussion of this can be found in Reference 2a.

In general, power spectral density analysis of a random function is required - - how to perform it becomes the next problem.

WAVE ANALYZERS

There are two basic types of analog analysis instruments for performing direct power spectral density analysis:

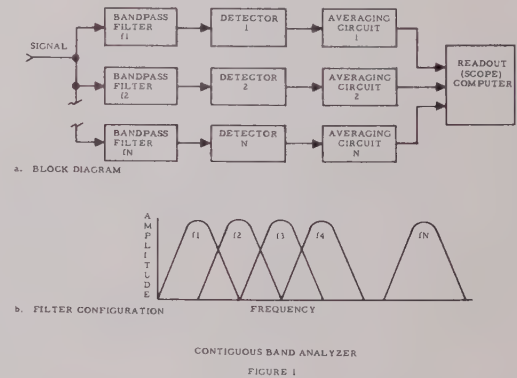
- (1) Contiguous band analyzers
- (2) Swept filter analyzers

The swept filter analyzer can be of two types:

- (a) Constant bandwidth
- (b) Constant percentage bandwidth

Contiguous Band Analyzer

The contiguous band analyzer is shown in Figure 1. The data signal is fed into the filters in parallel, with each filter center frequency fixed at a different frequency and generally such that adjacent filter skirts cross at the half power points.



The principal advantage of a contiguous band analyzer is the analysis speed. Since the spectrum of the input signal is broken down into as many parts as there are filters, then readout can be rapidly accomplished by commutating the output of each channel and displaying the commutated output on an oscilloscope, or going into a computer, etc.

Another advantage of the contiguous band analyzer is the ability to individually record the output of each channel vs. time, thus providing an analysis of non-stationary data signals.

The major disadvantages of the contiguous band filter analyzers are cost, sacrifice of filter quality in favor of cost considerations, and the lack of resolution; since to provide a contiguous band analyzer with high resolution would require a large number of filters. A minor problem of the parallel filter analyzer is the difficulty of resolving data that falls in the area of the filter crossovers.

These analyzers are used largely for quick look data and areas of interest are then analyzed with a narrow band swept filter analyzer.

Narrow Band Swept Filter Analyzers

(a) Constant Percentage Bandwidth Analyzer

The constant percent bandwidth analyzer gets its name from the fact that the filter bandwidth is a constant percentage of the filter's center frequency.

$$B = kf_0 \quad (7)$$

Where:

f_0 = analyzer center frequency

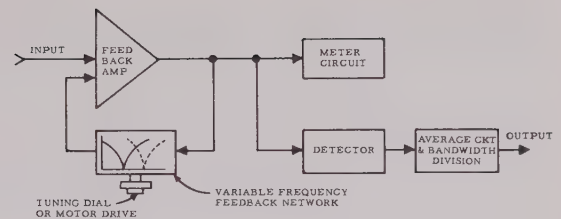
k = percent/100

Example: The bandwidth of a 10 percent bandwidth analyzer at a center frequency of 100 Hz is given by:

$$B = kf_0 = (.1)(100) = 10 \text{ Hz}$$

The most desirable feature of a constant percent analyzer is the provision of constant resolution. In analyzing data signals from structures, machinery, etc., where the resonant bandwidth and spectral components tend to become broader and further apart at higher frequencies, the constant percentage bandwidth analyzer provides optimum resolution and analysis speed.

The principal problem with the constant percentage bandwidth analyzers available is the relatively poor filter characteristics. This is brought about because the elements making up the filter have to be changed in order to change the center frequency and bandwidth continuously as the filter is swept through its frequency range. Another disadvantage is that the constant percentage bandwidth analyzer cannot be satisfactorily used for the analysis of periodic waveforms. A block diagram is shown in Figure 2.



CONSTANT PERCENTAGE BANDWIDTH ANALYZER
FIGURE 2

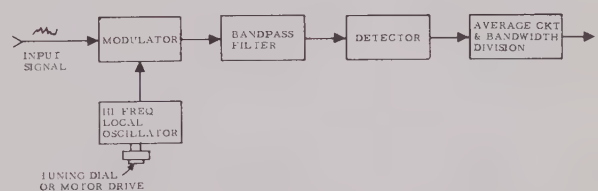
(b) Constant Bandwidth Analyzer

There are two basic types of heterodyne constant bandwidth analyzers:

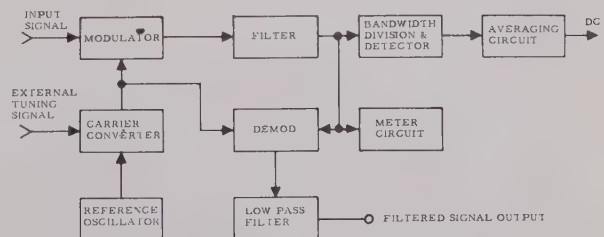
- (1) Tuned through the use of a high frequency local oscillator.
- (2) Heterodyne tracking analyzer using an external audio sine wave source for tuning. (Spectral Dynamics Model SD101A)

(There is a third type but it can be considered essentially the same as (1) above.)

A block diagram of a conventional heterodyne wave analyzer is shown in Figure 3, and a block diagram of the newer tracking analyzer is shown in Figure 4.



CONSTANT BANDWIDTH ANALYZER
USING LOCAL OSCILLATOR
FIGURE 3



CONSTANT BANDWIDTH TRACKING ANALYZER
FIGURE 4

The tracking analyzer in Figure 4 develops an equivalent signal of the heterodyne oscillator by a conversion of the audio signal into an upper sideband signal through modulation and phase cancellation of the unwanted sideband. (Reference 5) For simplicity the tracking analyzer can be considered a frequency tuned bandpass filter and will be represented in further block diagrams as shown in Figure 5. Figure 6 is a picture of a commercially available tracking filter/analyzer.

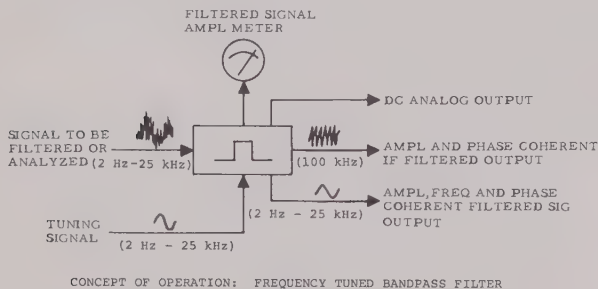
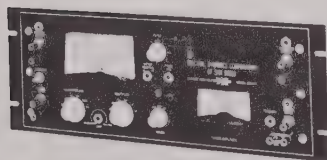


FIGURE 5



MODEL SD101A DYNAMIC ANALYZER TRACKING FILTER

FIGURE 6

Referring to Figure 5, the frequency tuned bandpass filter has a signal input, a tuning frequency input for selecting the center frequency of the filter and four outputs.

- (1) The filtered signal.
- (2) A d-c analog of the filtered signal amplitude.
- (3) A meter for indicating the amplitude of the signals within the filter passband.
- (4) A fixed frequency, 100 kHz signal proportional in amplitude to the filtered signals.

The principal advantage of the constant bandwidth heterodyne analyzer, whether it be (1) or (2) above, is that the data signals are selectively translated in frequency so as to fall within the fixed passband of a fixed center frequency filter. Since the filter center frequency and bandwidth are fixed, more engineering effort can be put into the design of the filter. This leads to better selectivity and steep-sided filter skirts approaching that of an ideal rectangular filter.

The principal disadvantage is the length of time required to perform an analysis.

WAVE ANALYSIS

Stationary Periodic & Complex Signal Analysis

The analysis of periodic or complex signals requires that the signal to be analyzed be long enough so that the swept filter analyzer will have enough time to sweep through the spectrum and display the sinusoidal components on a recorder. A typical analysis system block diagram for performing automatic sinusoidal or random analysis is shown in Figure 7. If the available source of signals is very short then, rather than perform an "on line" analysis, the data can be recorded and analyzed using a tape loop recorder.

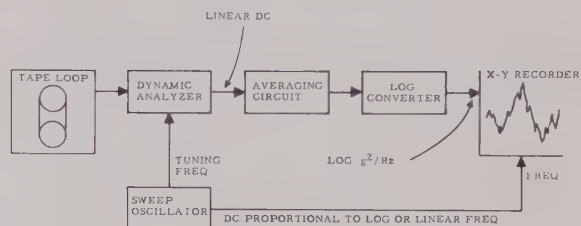


FIGURE 7

Record Length

The minimum length (sec) of a tape loop should be such that the analysis filter can properly respond and that there is enough time to have several periods of the lowest frequency data, say 10 periods.

$$T \geq 4/B \text{ seconds or,} \quad (8)$$

$$T \geq 10/f_1 \text{ whichever is greater} \quad (9)$$

Where:

T = Tape loop length in seconds
 B = 3dB bandwidth of analysis filter
 f_1 = lowest frequency to be analyzed

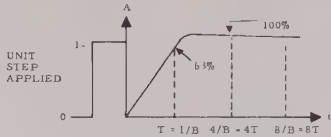
Equation 8 is based on the buildup time of a bandpass filter. The buildup time of a multiple-section crystal lattice filter with a shape factor of 4 has been determined empirically to be:

$$T \approx 1/B \quad (10)$$

Where:

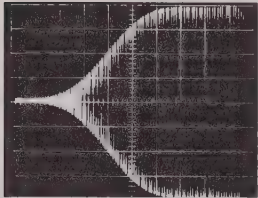
T = time required in seconds to reach 63% of the final output
 B = 3dB bandwidth of the filter

The filter reaches 98% to 99% of final output in approximately 4/B seconds. See Figure 8.



STEP RESPONSE OF MULTIPLE SECTION CRYSTAL FILTER WITH SHAPE FACTOR = 4

FIGURE 8(a)



TIME = 0.05 sec/cm
ACTUAL RESPONSE OF A 5Hz BANDWIDTH CRYSTAL FILTER

FIGURE 8(b)

FILTER RESPONSE

FIGURE 8

Thus, for the filter to properly respond, we should allow a buildup time of:

$$4/B \text{ seconds or } T_{100\%} \approx 4/B \text{ seconds} \tag{11}$$

Filter shape factor (S. F.) is defined as the ratio of the 60dB bandwidth to the 3dB bandwidth. (Figure 9)

$$S. F. = \frac{60\text{dB bandwidth}}{3\text{dB bandwidth}} \tag{12}$$

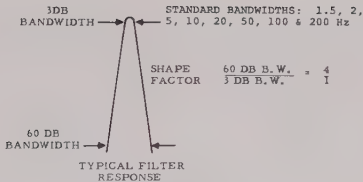
Then, for a shape factor of four, the 60dB bandwidth is simply 4 times the 3dB bandwidth. As an example, if a filter has a 3dB bandwidth of 2 Hz, the 60dB bandwidth of a filter with a shape factor of 4 would be 8 Hz.

It is interesting to note that a filter with a shape factor of four also corresponds to a filter roll off characteristic of 38dB per bandwidth. (Figure 9b)

What this means is that one bandwidth away from the filter cutoff frequency, the filter will be attenuating signals by 41dB; 2 bandwidths away from the center frequency, the filter will be attenuating a signal by 79dB, etc. to the ultimate attenuation level of the filter.

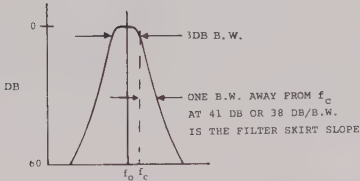
Averaging

In the remainder of this paper, averaging will be repeatedly mentioned. It is important to distinguish between true averaging and RC averaging.



FILTER SHAPE FACTOR

FIGURE 9(a)



FILTER SHAPE IN TERMS OF ATTENUATION BANDWIDTH OCTAVE

FIGURE 9(b)

CHARACTERISTICS OF SPECTRAL DYNAMICS CORPORATION
CONSTANT BANDWIDTH FILTER
FIGURE 9

True averaging is the arithmetic average of a signal over a time period T seconds long and the resultant value is divided by T to yield the true average. This type of averaging usually involves the use of an electronic integrator connected to the detector output of an analysis filter for a period of T seconds and the value at the output of the integrator is divided by T and then transferred to an X-Y recorder or other suitable readout device. The integrator is then reset to zero in preparation for reading the next data point.

$$\text{For true averaging: } \overline{f(t)} = \frac{1}{T} \int_0^T f(t) dt \tag{13}$$

Where: $\overline{f(t)}$ = average of any function of time

Averaging by use of a single section resistance and capacitance low pass filter is probably the most commonly used averaging circuitry because of the simplicity and the fact that it provides a continuous time averaged output as opposed to the true integrator which provides a sampled output-type display.

It is shown in Reference 1a that, after a signal has been applied to a single section RC averaging circuit for four or more RC time constants, the output becomes a time weighed function of all the preceding amplitudes with an equivalent averaging time of:

$$T = 2RC \tag{14}$$

Therefore, in all future expressions, T = 2RC can be used provided the detector signal has been applied for 4RC seconds or more.

Generally the analysis of periodic or complex signals requires little or no averaging following the

analyzer detector; however, it may be required on occasion. The output of the bandpass filter of a heterodyne type of analyzer is normally a signal in the 50 kHz to 100 kHz or higher region depending on the operating parameters associated with the analysis filter. Normally, the high frequency output from the filter is applied directly to a detector and averaging of the resulting d-c signal is minimal; on the order of $4/f_c$ to $10/f_c$ seconds to obtain an adequately pure d-c signal, where f_c is the center frequency of the analyzer filter. Therefore, the d-c analog output from this type of analyzer can rise virtually as fast as the filter response.

Example:

Suppose a heterodyne analyzer has a crystal filter operating at a center frequency of 100 kHz. The detector is seeing 100 kHz and the output of the detector is connected to an RC averaging circuit of say:

$$\frac{10}{f_c} = \frac{10}{100,000} = 0.0001 \text{ seconds}$$

Therefore, the dc output from the analyzer will follow the rise time of filters with bandwidths up to 1000 Hz. (Figure 8)

The point of this is to say that there is virtually no detector averaging time constant associated with a heterodyne analyzer when the detector is connected directly to the filter output.

If detection is accomplished on an audio filtered signal, the averaging time constant is a function of the desired degree of smoothness of the d-c output signal, the lowest frequency audio signal to be detected, and the type of detector, (i.e., mean², absolute average [full wave or half wave], peak, etc.) and the type of averaging circuit employed (i.e., true integrator averaging or RC circuit averaging).

If averaging is used following the detector, the buildup time of the analyzer final output to a read-out device would depend on the filter buildup time and the averaging circuit time constant.

Since the output of a single section RC circuit will reach 98% of full output in about $4RC$ time constants after the filter reaches 100%, then the buildup time is:

$$T \approx 4/B + 4RC \quad (15)$$

Where: B = the filter 3dB bandwidth and for true averaging:

$$T = 4/B + T_A$$

Where: T_A = Averaging time.

Detector

The type of detector makes little difference in the analysis of stationary or non-stationary periodic or complex signals since, for a properly resolved analysis, the signal from the analysis filter will be a sine wave and the output meter, recorder, etc., can be calibrated in terms of peak, average, or rms levels from the basic relationships:

$$\begin{aligned} 0.707 \times \text{peak} &= \text{rms} \\ 0.636 \times \text{peak} &= \text{average} \end{aligned}$$

Analysis Rate

Using no averaging, we would not want to scan the analysis filter any faster than one bandwidth for every $4/B$ seconds or:

$$AR \leq \frac{B}{4/B \text{ seconds}} = \frac{B^2}{4} \text{ Hz/second} \quad (17)$$

If we can scan any faster, errors due to filter blurring occur. If RC averaging is used, we would not want to scan any faster than one bandwidth for every $[4/B + 4RC]$ seconds or:

$$\frac{B}{\frac{4}{B} + 4RC} = \frac{B^2}{4(1 + BRC)} \quad (18)$$

This is because we have the filter buildup time plus the response of the RC circuit. If a true averaging circuit were used, Equation 18 would be reduced to:

$$AR \leq \frac{B^2}{4(1 + BT)} \quad (19)$$

$$\text{Total analysis time} = \frac{f_2 - f_1}{AR}$$

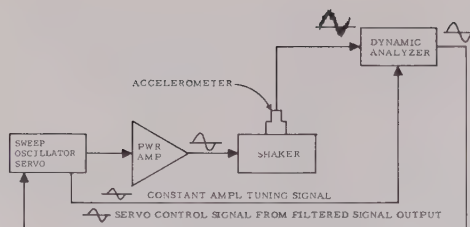
Where:

f_2 = highest frequency to analyze

f_1 = lowest frequency to analyze

Non-Stationary Periodic and Complex Signal Analysis

The analysis of this type of signal by a narrow-band analyzer until recent years was very difficult. With the development of the frequency tuned band-pass filter (Figure 5) or tracking filter as it is sometimes called, this becomes possible. One simple example of a non-stationary complex signal is a swept sine wave test of an object using an electromagnetic or hydraulic vibrator. A block diagram of a typical system set-up is shown in Figure 10.

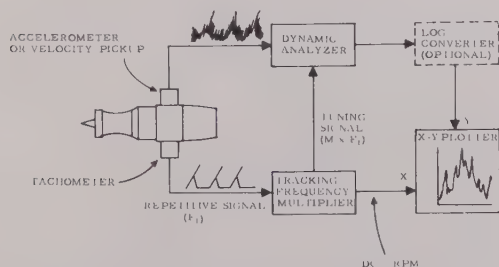


SWEPT-SINE VIBRATION TESTING
FIGURE 10

In this case, it is desired to analyze the fundamental structural responses of the test package at several locations. The response signals can be anything but pure sinusoids if the package has rattling components and other non-linearities. The method of analysis shown is to connect the response pickup/s into the frequency tuned bandpass filter and tune the filter with the output of the swept sine wave oscillator used to drive the shaker. Thus, the center frequency of the bandpass filter is continuously tuned to pass only the fundamental response continuously rejecting noise and harmonics.

If it is desired to track the harmonics of the response signal, a device for converting the fundamental of the sine wave oscillator output used for tuning the tracking analyzer to the desired harmonic is required. These devices are available.

Another example is the analysis of vibrating shafts of a rotating machine. (Figure 11.) Using the tracking analyzer tuned from a sine wave tachometer or from a pulse tachometer suitably converted to a sine wave or from a frequency multiplier producing the desired shaft frequency, the tracking analyzer can produce plots of shaft vibration vs. speed. Such converters and frequency multipliers are available. (Reference 6.)



TRACKING VIBRATION ANALYSIS OF SPECIFIC ROTATION COMPONENTS
FIGURE 11

The analysis of quasi-sinusoids and non-stationary complex signals without having some reference

signal (such as excitation source, tachometer, etc.) becomes very tedious unless special signal tracking filters are used or other special techniques are used (Reference 2b.)

The maximum analysis rates are dependent upon the filter bandwidth, the specimen and other equipment limitations and has to be evaluated for a particular set of circumstances.

However, it should be noted that a basic tracking analyzer with an infinitely fast tracking rate still imposes the restriction that the maximum rate a step change in amplitude can be properly resolved is limited to the filter buildup time. (Equation 11.)

Resolution

The proper resolution for analyzing periodic signals is a filter bandwidth that will separate the nearest suspected component. For a distorted sinusoid in which the harmonics are related, this becomes:

$$B_{\max} = f_1 \quad (20)$$

$$B_{\min} = f_1/2 \quad (21)$$

Where:

f_1 = fundamental frequency and

B = 3 dB bandwidth of analysis filter

With analysis filter shape factors of 4 or better, $B = f_1$ will provide approximately 30 dB attenuation to adjacent harmonic frequency components during the analysis.

$B = f_1/2$ will provide approximately 60 dB attenuation to adjacent harmonic components.

It is of note to point out that, since the harmonics of periodic signals are f_1 Hz apart, a constant percentage bandwidth analyzer would not be suitable for this type of analysis.

Transient Signal Analysis

The normal method of analyzing a transient is to make the transient repetitive through the use of a tape loop, magnetic drum, or other suitable device and perform the analysis as if it were a periodic function with a fundamental frequency equal to the repetition rate of the transient.

Making the signal repeat produces a Fourier series which can then be integrated electrically or manually to produce the pulse spectrum.

$$\text{Number of spectral lines between amplitude nulls in the pulse spectrum} = T_c/D \quad (22)$$

Where:

T_C = Repetition rate in seconds/cycle

D = Duration of the transient in seconds

A more detailed description of the techniques and some results of doing pulse analysis with heterodyne analyzers can be found in Reference 3. The resolution and analysis rate considerations are the same as those for periodic analysis. The minimum record length for transient analysis is shown in Reference 3 to be:

$$T_r(\text{min}) = 2D \quad (23)$$

Where:

$T_r(\text{min})$ = minimum record length in seconds

Stationary Random Signal Analysis

To understand the analysis of random signals, it is important to understand a few of the basic fundamentals of random signals.

First, it must be understood that the amplitude of a random signal never repeats exactly; therefore, from a mathematical standpoint the instantaneous amplitude at any future time cannot be predicted. The only thing that can be said about a future occurrence is that a signal within certain amplitude limits will occur with a certain probability.

The method of displaying this probability of occurrence is the amplitude probability density curve and gives a description of the amplitude characteristics of the random signal.

For a stationary random signal, the probability density function $p(y)$ is as follows (from Reference 1b):

$$p(\chi) = \frac{1}{T\Delta\chi} \sum_{n=1}^{n=\infty} t_n(\chi, \chi + \Delta\chi) \quad (24)$$

$$\lim_{\substack{T \rightarrow \infty \\ \Delta\chi \rightarrow 0}}$$

The quantity t_n is the time spent by the amplitude within the narrow range of χ and $\chi + \Delta\chi$, T is the total observation time.

Figure 12 shows diagrammatically what this means. If we had an amplitude window that was χ units high and $\Delta\chi$ wide and we measured the average time spent by the signal within the window for various levels or height of χ then we could describe the amplitude probability density of the signal.

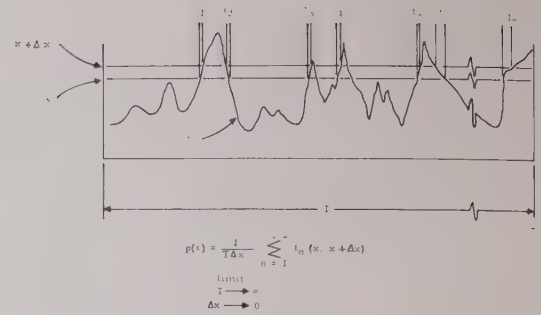


ILLUSTRATION OF THE PROBABILITY DENSITY FUNCTION $p(x)$ FOR A STATIONARY RANDOM SIGNAL

FIGURE 12

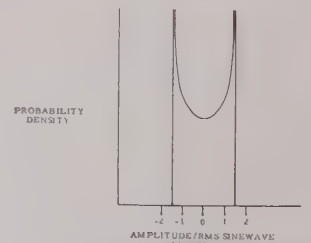
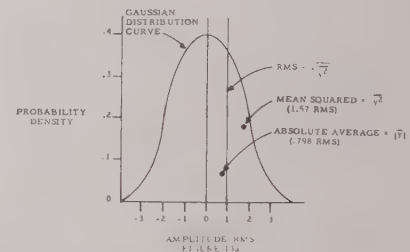
The most common amplitude probability density curve found in nature is the Gaussian, or "Normal" curve.

$$\text{Given by: } p(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \quad (25)$$

Where σ is the rms level about the mean μ , σ^2 is referred to as the variance and is equal to the mean square value if the mean is zero, (which is the usual case). Thus with an rms level equal to 1 and a mean value equal to zero, Equation 25 reduces to:

$$p(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \quad (26)$$

Some typical probability density curves are shown in Figure 13.



TYPICAL PROBABILITY DENSITY CURVES
FIGURE 13

The interpretation of the probability density curve may be best understood by making a reading from the curve.

- (1) At an amplitude of zero, the probability density curve tells us that 39% of the time the amplitude will be in the vicinity of zero.

- (2) At an amplitude of 1, or the rms value, the probability density curve reads that 24% of the time the amplitude will be in the vicinity of one. (Note that the curve is normalized to an rms value equal 1.)

The total area under the probability density curve is equal to 1 since we are certain that there will be some amplitude between $\pm$ infinity at any time of observance. A further discussion of this can be found in Reference 1c and 8a.

Just as the average, rms, and mean square relationship of a known function (such as a sine wave, square wave, pulse, etc.) can be determined in the time domain, so can the average, rms, and the mean square values of random signals be determined in the amplitude domain from the amplitude probability density curve. (Reference 8a.)

$$\text{mean, } \bar{y} = \int_{-\infty}^{\infty} yp(y) dy \quad (27)$$

$$\text{mean square, } \overline{y^2} = \int_{-\infty}^{\infty} y^2 p(y) dy \quad (28)$$

$$\text{the absolute average, } |y| = \int_{-\infty}^{\infty} yp(y) dy \quad (29)$$

To find the relationship of the absolute average value to the rms value of a random signal with a Gaussian probability density with the mean value equal to zero, substitute Equation 25 into Equation 29. (Reference 2c.)

$$|y| = \frac{2}{\sqrt{2\pi}\sigma} \int_0^{\infty} e^{-\frac{y^2}{2\sigma^2}} y dy = \sqrt{\frac{2}{\pi}} \sigma \quad (30)$$

Where: σ = rms value

Some other relationships for Gaussian distribution of note can be derived from Equation 30.

$$\text{absolute average, } |y| = \sqrt{2/\pi} \sigma = 0.798 \times \text{rms} \quad (31)$$

$$\text{mean square, } \overline{y^2} = 1.57 |y|^2 \quad (31a)$$

$$\text{rms} = 1.25 |y| \quad (31b)$$

Reviewing the relationship of a sine wave:

$$\text{rms} = 0.707 \times \text{peak} \quad (32)$$

$$\text{absolute average} = 0.636 \times \text{peak} \quad (33)$$

then,

$$\text{rms} = \frac{0.707}{0.636} \times \text{average} = 1.11 \times \text{average} \quad (34)$$

Now we note that, if the readout of an analyzer is based on detecting the absolute average of a signal but is calibrated for rms readout of sine waves, to use the same detector for reading out the rms value of a Gaussian Random Signal, the following relation exists.

$$\frac{\text{rms}_{(\text{random})}}{\text{rms}_{(\text{sine})}} = \frac{1.25 |y|_{\text{random}}}{1.11 |y|_{\text{sine}}} = 1.13 \quad (35)$$

Therefore, to calibrate the output of an analyzer with an absolute average detector to read the rms level of Gaussian distributed random noise and using a sinewave calibration source, the rms meter indication for Gaussian noise applied to the detectors will be 1.13 times higher than the corresponding rms value of a sine wave applied to the detector. To state this simply, the rms sine wave calibration point must be multiplied by 1.13 to indicate the correct level of a Gaussian random signal of the same rms value.

These associations can be applied to the calibration of a wave analyzer employing an averaging detector for power spectral density analysis.

The mean value, mean square value, and rms value of a random signal can be computed from the probability density function (Reference 2c and 8a). The same quantities are computed for periodic functions or random functions in the time domain from:

$$\text{Mean value, } \bar{y} = \frac{1}{T} \int_0^T y(t) dt \quad (36)$$

Where:

T = 1/frequency for a periodic function

$y(t)$ = some function of time

$T \rightarrow \infty$ for a random function

similarly for the mean square,

$$\overline{y^2} = \frac{1}{T} \int_0^T y^2 dt \quad (37)$$

The rms value is equal to:

$$\text{rms} = \sqrt{\overline{y^2}} \quad (38)$$

POWER SPECTRAL DENSITY ANALYSIS

Just as the amplitude characteristics of a random signal are described by its probability density function in the amplitude domain and in the time domain by infinite averages of its absolute mean, mean square, and rms values, the spectral characteristics of a random signal are described by the measurement of its power spectrum as defined by: (Figure 14)

$$Py(f) = \frac{1}{T} \int_0^T \frac{Y_B^2}{B} dt \quad (39)$$

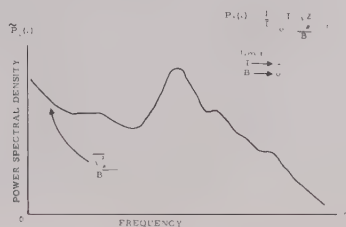
limit

$T \rightarrow \infty$

$B \rightarrow 0$

Where:

- $P_y(f)$ = Power spectral density function
 y^2_B = Instantaneous square of the signal within a narrow bandwidth (B).
 B = Signal bandwidth



TYPICAL POWER SPECTRUM
FIGURE 14

This says that the true power spectral density function is obtained by averaging the mean square value of a signal in a narrow bandwidth B cycles wide over an infinite time period and with a filter bandwidth infinitely small.

Since in practice we don't have infinitely long random records nor the time to average to infinity and don't have infinitely narrow bandwidths, we must simply estimate the power density function from a finite length of record, averaging over finite times and with finite bandwidths. Thus, Equation 39 becomes:

$$P_y(f) = \frac{1}{T} \int_0^T \frac{y^2_B}{B} dt = \frac{\overline{y^2_B}}{B} \quad (40)$$

$P_y(f)$ means an estimate of the power density
 T is a finite averaging time
 B is the effective noise bandwidth of the analysis filter

$\overline{y^2_B}$ is the average of the square of the amplitudes within the analysis bandwidth

The above equation assumes a flat spectrum within the passband of the analyzer. The noise bandwidth is defined as the bandwidth of an ideal rectangular sided filter which would pass the same amount of signal power as the real filter with the input spectrum flat (white). White noise is defined as a random signal with a uniform power spectral density function.

The noise bandwidth of a filter can be determined empirically by several methods but is usually determined by plotting the square of its voltage transfer function, determining the area under the resulting plot and constructing a rectangle the same height containing the same area. A typical plot is shown in Figure 15.

The noise bandwidth can also be determined by the use of a noise generator, band limited, and producing a flat power spectrum. The total mean square output level from a band limited source can be obtained from:

$$\overline{y^2} = \int_{f_1}^{f_2} \frac{y^2_B}{B} df \quad (41)$$

where $\overline{y^2}$ is the expression for the area under the power spectrum curve, which f_1 and f_2 are the lowest and highest frequencies of interest.

For a flat power spectrum,
 Equation 41 reduces to:

$$\overline{y^2} = \frac{y^2_B}{B} (f_2 - f_1) \quad (42)$$

This equation leads to one of the most generally used expressions by environmental engineers. By taking the square root of both sides of Equation 42

$$\sqrt{\overline{y^2}} = \sqrt{\frac{y^2_B}{B} (f_2 - f_1)} \quad (43)$$

or

$$(\text{rms}) = \sqrt{\text{PSD} \times B} \quad (44)$$

Where:

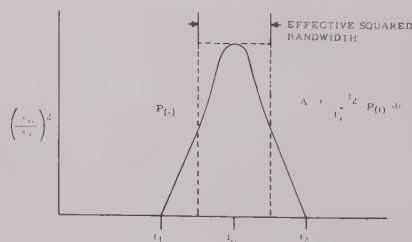
PSD = power spectral density, usually expressed in units of g^2/Hz

B = effective noise bandwidth of a random signal being investigated

NOTE: The half power (3dB) bandwidth of filters with a shape factor of four or less for all practical purposes is the same as the noise bandwidth. Thus, to perform a power spectral density analysis you must perform the following:

- Filtering the random signal with a bandpass filter.
- Detecting the output of the filter with a linear or square law detector.
- Averaging the detected quantity.
- Divide by the noise bandwidth of the filter.

For contiguous band analyzers this means "n" filters, "n" detectors, "n" averaging circuits, bandwidth division can be taken care of as a calibration constant.



- PLOT THE SQUARED RESPONSE CHARACTERISTICS OF THE FILTER ON LINEAR PAPER.
- INTEGRATE THE CURVE TO FIND THE TOTAL AREA.
- CONSTRUCT A RECTANGLE OF THE SAME HEIGHT CONTAINING THE SAME AREA. THE WIDTH OF THE RECTANGLE IS THE EFFECTIVE SQUARED BANDWIDTH.

CALCULATION OF EFFECTIVE SQUARED BANDWIDTH
FIGURE 15

For the constant percent swept filter analyzer, the operation of (a) through (d) are required, and (d) is required on a continuously variable basis. For the constant bandwidth analyzer, operations (a) through (c) are required and (d) can be accomplished as a calibration constant unless the constant bandwidth analyzer offers a switchable or automatic selection of filter bandwidths.

As the center frequency of the analysis filter is swept through the frequency range of interest, the output is usually recorded vs. frequency, thus providing a continuous plot of the power spectral density estimate.

Detectors

From the foregoing discussions, it would appear that the proper detector for a power spectral density analyzer would be a square law or analog squaring circuit. These types of detectors are preferred except that they are difficult and costly to build and suffer a lack of continuous dynamic range. By comparison, the linear absolute average detector is simple to build, calibrate, and has very wide dynamic range. The dynamic range consideration is probably the most important consideration for the construction of automatic power density analyzers.

Referring back to Equation 31a

$$\overline{y^2} = \frac{\pi}{2} |y|^2 \quad (45)$$

Where:

$\overline{y^2}$ = the average of the instantaneous square of y, and
 $|y|^2$ = the square of the averaged absolute value, which is the relationship of the average absolute value to the mean square value of a random signal with a Gaussian Probability Density.

Since the power spectral density function was given from Equation 39 to be:

$$Py(f) = \frac{1}{T} \int_0^T \frac{\overline{y^2}_B}{B} dt = \frac{\overline{y^2}_B}{B} \quad (46)$$

lim
 $T \rightarrow \infty$
 $B \rightarrow 0$

Where:

$\overline{y^2}_B$ = the square of the instantaneous level in a bandwidth B.
 $\overline{y^2}_B$ = the average of the instantaneous squared levels in a bandwidth B (mean squared).

Then, the power density function can be modified by the relationship of the average absolute value, $|y|^2$ to the mean squared value $\overline{y^2}$ such that:

$$Py(f) = \frac{\pi}{2B} |y|^2 \quad (47)$$

Where:

$|y|^2_B$ = The average absolute value squared in a bandwidth B.

Thus, by appropriate readout scales, the power density can be read out with an absolute average detector or a log converter can be used to read out the square of the rms level by adjustment of the readout sensitivity.

There has been much discussion over the use of the averaging detector in power spectral density analyzers. Of most concern in using this type detector is the departure of the random signal applied to the detector from a Gaussian distribution.

For a properly resolved power density measurement and due to the central limit theorem of statistics, the departure from Gaussian distribution of the output of a narrow bandwidth filter with a random signal applied appears to be so small as to be insignificant, seemingly regardless of the input probability density.

The central limit theorem (Reference 1d and 4) states essentially that the distribution of statistically independent samples tend toward a Gaussian distribution regardless of the probability density of the process being sampled. The statistically independent sample can be considered the output of a narrow band filter. The assumption of a Gaussian distribution appears valid if the bandwidth of the filter is approximately equal to or less than the bandwidth of the resonance producing the non-Gaussian probability distribution. Empirical data taken from filtering different bandwidth of non-Gaussian noise through various filter bandwidths are shown in the Appendix. This empirical data is discussed in Appendix I.

Statistical Accuracy

Since we are just providing estimates of the power spectrum, it would be of interest to know the probable range of error in our estimate.

Assuming perfect instruments, there is a statistical error associated with the output of any power density analyzer.

This error is a function of the total number of samples available on which to base the estimate and is related to the record length, analysis filter bandwidth, and the averaging time. This error can be expressed in terms of the standard deviation of the measured power density function or:

$$\sigma_{Py(f)} \sim \frac{Py(f)}{\sqrt{BT}} \quad (48)$$

(See Reference 1e.)

Where:

$\sigma \tilde{P}_y(f)$ = the standard deviation of the measured power spectrum

$P_y(f)$ = the true power spectrum

B = filter noise bandwidth

T = averaging time

Let the ratio of

$$\frac{\sigma \tilde{P}_y(f)}{P_y(f)} = \epsilon = \frac{P_y(f)}{\sqrt{BT}} = \frac{1}{\sqrt{BT}} \quad (49)$$

Then $\epsilon \approx \frac{1}{\sqrt{BT}}$

What this means is that for a given measurement of the power spectral density, the error (ϵ) between the measured power density and the true power density will be $\pm \epsilon$ with a confidence factor of 67%, or two-thirds of the measurement will be within $\pm \epsilon$. This formula only holds for $\epsilon < 0.2$. If ϵ becomes larger than 0.2 or $\pm 20\%$, the deviation about the true measurement takes on a different distribution called the chi-squared distribution (X^2) and, if desired, can be used entirely in place of Equation 49 above.

If the random amplitudes on the output of an analyzer are Gaussian, then a mean square estimate $\frac{\tilde{y}^2}{2}$,

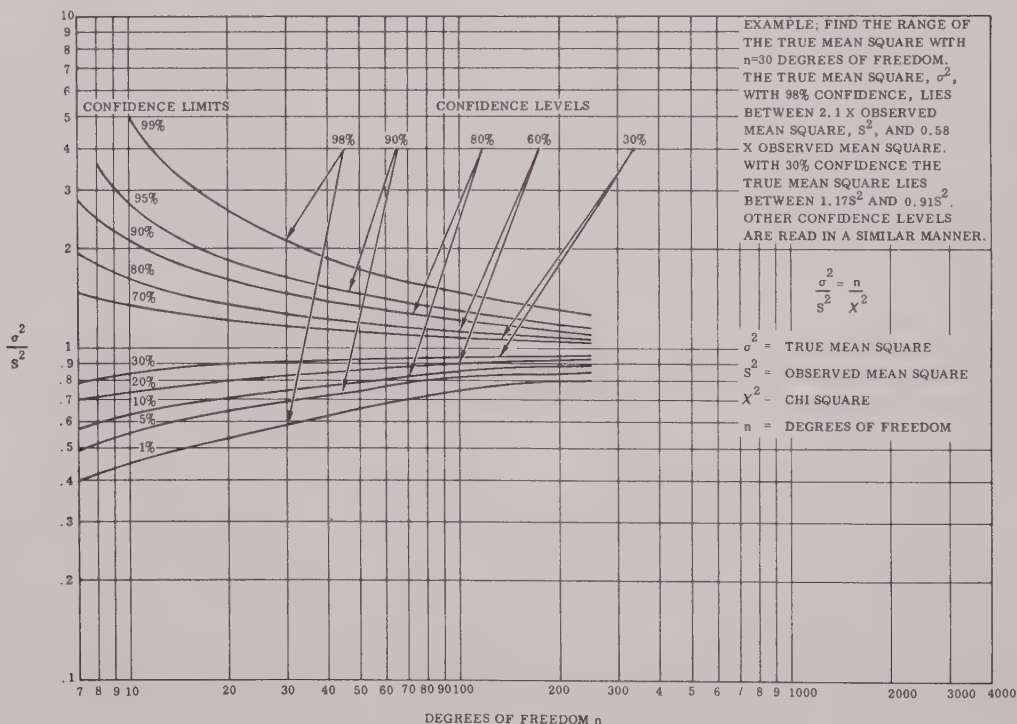
obtained will be distributed about the true mean square value, y^2 , with a chi-square distribution of "n" degrees of freedom.

Mathematically this is expressed by:

$$\frac{\tilde{y}^2}{y^2} = \frac{X^2}{n} \text{ or } y^2 = \left(\frac{n}{X^2} \right) \tilde{y}^2 \quad (50)$$

The value of X^2 for various confidence limits can be obtained from standard tables. (Reference 1f.)

A limited chart of the X^2 distribution with "n" up to 250 is given in Figure 16, and examples are given on the chart; however, some additional explanation can help. Observing the chart the reader will notice curves labeled confidence limits and then arrows denoting confidence levels. The upper limits (above 1) mean that Y percent of the time the true mean square value will be below that limit line and the lines below (1) mean that Y^1 percent of the time the true mean square level will be below the limit line, therefore, we can say with $Y-Y^1$ percent confidence that the true mean square level will lie between Y and Y^1 .



CHI-SQUARE DISTRIBUTION

FIGURE 16

It has been shown in Reference 1g that:

$$n = 2BT \quad (51)$$

Where:

B = the noise bandwidth of the filter
T = averaging time

Now, where:

$n = 2BT$
T = true averaging time

Now, let:

$$T = 2RC \quad \text{then,} \quad n = 4BRC \quad (52)$$

Therefore, it is desirable to maintain as high a BT product as practical. The wider the bandwidth and the longer the averaging time, the less data scatter (statistical error). If we are to perform analysis from a time limited source of random data such as from a tape loop, then the maximum statistical confidence available from the record is:

$$n = 2BT_r \quad (53)$$

Where:

T_r = the record length in seconds

To preserve "n" available from the record, we should average the output of an analysis filter for T_r seconds, in other words in Equation 53, let $T = T_r$.

Averaging longer than T_r does not improve the statistical confidence of the measurement.

In summary, the statistical error in percent for an estimate of the power density may be expressed, using the following formula:

$$100\epsilon = \frac{1}{\sqrt{BT}} = \frac{1}{\sqrt{2BRC}} \quad (54)$$

Where:

100ϵ = % error with 67% confidence and ϵ is restricted to $\epsilon \leq 0.2$.

T = true averaging time (sec.)

RC = time constant (sec.)

(R is usually expressed in megohms and C in microfarads.)

Example:

RC time of a circuit with 25,000 ohms and 10 mfd.

$T = (.025 \text{ meg})(10 \text{ mfd}) = .25 \text{ sec.}$

or for determination of statistical errors using the chi-square distribution tables.

$$n = 2BT \quad \text{or} \quad n = 4BRC \quad (\text{from Equations 51 and 52})$$

Maximum available statistical confidence levels available from a sample recording of a random signal of length T_r (sec)

$$n_{\max} = 2BT_r = 4BRC \quad (57)$$

$$\text{Where:} \quad RC \geq \frac{T_r}{2}$$

$$100\epsilon_{\min} = \frac{1}{\sqrt{BT_r}} = \frac{1}{\sqrt{2BRC}} \quad (56)$$

also, ϵ is also related to n by $\epsilon = \frac{1}{\sqrt{n/2}}$

Resolution Considerations

In the analysis of the power spectrum from actual structures, it can be seen from the previous discussions that the statistical quality of measurement improves as the bandwidth of the analysis filter is broadened and the averaging time is increased. However, the analysis time suffers with increases in averaging time and the ability to resolve resonances and peaks in the power spectrum suffers with wide filter bandwidths. Therefore, the selection of the proper bandwidth for an analysis becomes somewhat of an art. To optimize the analysis time and resolution, you must virtually know the results of an analysis before you analyze. Quite often this is resolved by making quick-look scans of the data prior to performing the final analysis.

Let us return to the discussion earlier regarding the constant bandwidth analyzer vs. the constant percentage bandwidth analyzer. It was stated earlier that the resolution of a constant bandwidth analyzer increased with frequency, whereas the resolution of a constant percentage analyzer is constant, where:

$$\% \text{ resolution} = \frac{B}{f_0} \times 100 \quad (57)$$

Where:

f_0 = analysis filter center frequency

B = analyzer bandwidth

It can be seen that, for a constant bandwidth analyzer, the percent resolution decreases inversely with the center frequency and for the constant percentage analyzer, we can replace B with kf , where k is a fixed percentage of the center frequency of the analysis filter. Thus:

$$\% \text{ resolution} = \frac{kf_0}{f_0} = k \quad (58)$$

In general, the responses of real structures tend to be constant Q in nature, that is, the resonant bandwidths of the structural resonances tend to be broader at higher frequencies. This would lead us to the conclusion that optimum resolution of a power spectrum would be obtained using a constant percentage analyzer.

It has been suggested (Reference 1h) that a filter bandwidth of 1/4 the half power bandwidth of the narrowest resonant peak be used as a general guide; this, however, should set only the minimum resolution required. (The highest % resolution)

Since:

$$B \leq \frac{B_8}{4} \quad (59)$$

Where:

B = analysis bandwidth
B₈ = half power bandwidth of narrowest resonance or peak in the power spectrum

Then maximum % resolution,

$$p_{\max} = \frac{B_8}{4 \times F_8} \times 100 = \frac{B_8}{4F_8} \times 100 \quad (59a)$$

Where:

F₈ = the center frequency of the resonance or peak in the power spectrum

This gives an equivalent expression related to the maximum Q of a structure:

$$Q_{\max} = \frac{F_8}{B_8} \quad (60)$$

Therefore, the Q_a of a constant % (or constant Q) analyzer can be related to the Q_{max} of the spectrum by,

$$Q_a = 4Q_{\max} \quad (61)$$

Where:

Q_a = required selectivity of a constant percentage bandwidth analyzer, expressed by:

$$Q_a \geq \frac{4F_8}{B_8} \quad (62)$$

Analysis Rate and Total Analysis Time

The next questions are (a) how fast can we sweep a constant bandwidth filter and a constant percentage bandwidth filter, and (b) how much time will it take to perform a power spectral density analysis for each.

Analysis Rate of a Constant Bandwidth Analyzer

If the filter buildup time (Equation 11) $T_{100\%} \approx 4/B$ seconds and we then follow the detector with a true averaging circuit, we wouldn't want to move the filter any faster than one bandwidth for every 4/B seconds, and then we would only desire an average value of the power from the filter after this time, otherwise our data will be somewhat influenced by the transient signals during the buildup time. An expression for this is:

$$AR \leq \frac{B}{\frac{4}{B} + T} \quad (63)$$

Where:

AR = analysis rate in Hz/sec
B = filter noise bandwidth in Hz
T = averaging time in seconds

Further examining the above equation and considering the practical values of T for a given B based on desirable statistical confidence levels, we see that the 4/B term will make very little difference in the final scan rate value. In other words, as B becomes large, T becomes smaller (usually) and the 4/B part becomes negligible compared to T, and for a small B, T becomes large.

Therefore, Equation 63 becomes:

$$AR \leq \frac{B}{T} \quad (64)$$

If RC averaging is used, then we must allow at least 4RC time constants before the output of the averaging circuit is providing a correct averaged value. Therefore, we wouldn't want to sweep the analyzer filter any faster than one bandwidth for every 4RC time constants or:

$$AR \leq \frac{B}{4RC} \quad (65)$$

Let's now compare Equations 64 and 65. Remembering that after 4RC seconds that $RC = T/2$. AR for true averaging with the equivalent RC time constant is twice that of an analyzer using RC averaging from:

$$AR_T \leq \frac{B}{2RC} \quad (\text{using Equation 64})$$

Where:

AR_T = analysis rate using true averaging

and,

$$AR_{RC} \leq \frac{B}{4RC} \quad (\text{using Equation 65})$$

Where:

AR_{RC} = analysis rate using RC averaging

Therefore: $\frac{AR_T}{AR_{RC}} = \frac{B/2RC}{B/4RC} = 2$

This becomes a consideration when trying to shorten the analysis time. It is also interesting to note that the analysis rate is related to the statistical confidence levels:

Since $n = 2BT$ or $B = \frac{n}{2T}$ and using Equation 64,

$$AR \leq \frac{B}{T}, \text{ we find } AR(T) = B = \frac{n}{2T} \text{ or}$$

$$n = 2AR(T)^2 \quad (66)$$

Analysis Rate and Time for Constant Bandwidth Analyzer

The above analysis rates state that the analysis filter should be scanned using a linear sweep rate, or:

$$\frac{df}{dt} = AR \quad (67)$$

then $\int_{f_1}^{f_2} df = AR \int_0^{A_t} dt$, solving $t \Big|_0^{A_t}$ provides

$$A_t = \frac{1}{AR} (f_2 - f_1) \quad (68)$$

Where: A_t = total analysis time (sec.)
 f_2 = highest analysis frequency (Hz)
 f_1 = lowest analysis frequency (Hz)
 AR = analysis rate (Hz/sec.)

Then the total analysis time is equal to the total frequency range divided by the analysis rate.

A sample problem of "on-line" analysis using the equations developed for a constant bandwidth analyzer follows:

1. Desire $\pm 10\%$ statistical accuracy;
 $10\% = 100\epsilon$ (from Equation 54)
2. Analysis frequency range, 10 Hz to 2000 Hz
3. Analyzer bandwidth (B) required for desired resolution is 10 Hz (noise bandwidth)

Since the error desired is less than 20%, we can use:

$$\epsilon = \frac{1}{\sqrt{BT}}, \quad BT = \frac{1}{\epsilon^2} = \frac{1}{.01} = 100$$

Since $B = 10$, $T = \frac{100}{10} = 10$ second averaging time.

If the analyzer uses RC averaging, then:

$$\epsilon = \frac{1}{\sqrt{2BRC}}, \quad RC = \frac{1}{2\epsilon^2 B} = 5 \text{ seconds}$$

$$AR = \frac{B}{T} = \frac{10}{10} = 1 \text{ Hz/sec (for true averaging)}$$

$$AR = \frac{B}{4RC} = \frac{10}{20} = 0.5 \text{ Hz/sec (for RC averaging)}$$

$$A_t = \frac{2000 - 10}{AR} = \frac{1990}{1}$$

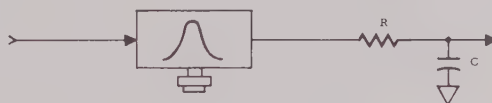
= 1990 seconds or 33 minutes (for true averaging)

$$A_t = \frac{2000 - 10}{AR} = \frac{1990}{0.5}$$

= 3980 seconds or 66 minutes (for RC averaging)

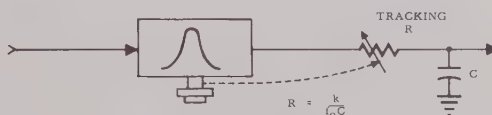
ANALYSIS RATE AND TIME FOR CONSTANT PERCENTAGE BANDWIDTH ANALYZER

Let us now examine the analysis rate and time, statistical confidence levels for a constant percentage bandwidth analyzer using fixed averaging time (Figure 17a).



CONSTANT PERCENTAGE BANDWIDTH ANALYZER WITH FIXED RC AVERAGING

FIGURE 17(a)



CONSTANT PERCENTAGE BANDWIDTH ANALYZER WITH TRACKING RC AVERAGING

FIGURE 17(b)

CONSTANT PERCENTAGE BANDWIDTH ANALYZERS USING "FIXED" AND "TRACKING" RC AVERAGING

FIGURE 17

Since the bandwidth is a percentage of the center frequency, let:

$$B = pf_0 \quad (69)$$

Where: p = percentage bandwidth/100
 f_0 = analysis filter center frequency
 B = analyzer noise bandwidth

Then substituting Equation 69 in Equation 51 and 52:

$$n = 2pf_0 T \text{ (for true averaging)} \quad (70)$$

and

$$n = 4pf_0 RC \text{ (for RC averaging)} \quad (71)$$

This shows that the statistical confidence level increases directly with the center frequency of the analyzer.

Now let's look at the analysis rates.

$$AR \leq \frac{pf_0}{T} \quad (72)$$

or

$$\frac{df}{dt} = \frac{pf_0}{T} \quad (73)$$

This says that the scan rate is a first order function of the analysis frequency or is a logarithmic sweep and each decade will be swept in the same time.

Solving the differential equation for the total analysis time (A_t) over a frequency range of f_1 to f_2 :

$$\int_{f_1}^{f_2} \frac{df}{f} = \frac{p}{T} \int_0^t dt, \quad \frac{T}{p} \ln f \Big|_{f_1}^{f_2} = t = A_t$$

Therefore, the total analysis time:

$$A_t = \frac{T}{p} \ln \frac{f_2}{f_1} \quad (74)$$

Example: assume--

1. 10% bandwidth analyzer
2. Frequency range of analysis from 10 Hz - 2000 Hz
3. Averaging time $T = 10$ seconds
4. "On-line" analysis

$$p = \frac{10\%}{100} = 0.1$$

$$\text{Total analysis time } A_t = \frac{10}{0.1} \ln 200 = (100)(5.3)$$

$$= 530 \text{ sec or } 8.8 \text{ minutes.}$$

The statistical confidence ranges from (Equation 70) $n = (2)(.1)(10)(10) = 20$ degrees of freedom at 10 Hz to $n = (2)(.1)(2000)(10) = 4000$ degrees of freedom at 2000 Hz.

It would appear that some analysis time improvement could be achieved if we had an averaging circuit that also changed value with frequency, something similar to a frequency tracking RC

circuit (Figure 17b). Then we could have not only a constant resolution, but a constant statistical accuracy at all analysis frequencies.

For $n = 4BRC$, $B = pf$ (using Equation 69)

$$\text{Let } RC = \frac{k}{f_0} \quad (75)$$

Where:

$k =$ a constant depending upon the desired statistical confidence level

$f_0 =$ analyzer filter center frequency

Then,

$$n = \frac{4pf_0k}{f_0} = 4kp \quad (76)$$

Thus, with a controlled RC, we have a constant statistical confidence level since p and k are both constants for any given set of conditions.

$$\text{The analysis rate is: } AR = \frac{B}{4RC} = \frac{pf}{4k} = \frac{pf_0^2}{4k}$$

This says the analysis rate is a second order function of frequency.

Since,

$$AR = \frac{df}{dt} = \frac{pf^2}{4k} \quad (77)$$

Then,

$$\int_{f_1}^{f_2} \frac{df}{f^2} = \frac{p}{4k} \int_0^{A_t} dt; \quad -\frac{1}{f} \Big|_{f_1}^{f_2} = \frac{p}{4k} t \Big|_0^{A_t}$$

$$\text{or total analysis time } A_t = \frac{4k}{p} \left[\frac{1}{f_1} - \frac{1}{f_2} \right] \quad (78)$$

Example: let's assume:

1. Tape loop length = 10 sec
2. n desired = 100
3. Analysis frequency range = 10 Hz-2000 Hz
4. 10% analysis bandwidth

$$\text{Then, } p = \frac{10\%}{100} = 0.1, \quad n = 4kp, \quad k = \frac{n}{4p} = \frac{100}{0.4} = 250$$

$$\text{Using Equation 75, } RC = \frac{k}{f_0} = \frac{250}{f_0}$$

This means at say $f_0 = 100$ Hz, we would adjust our tracking RC circuit to a value of $\frac{250}{100} = 2.5$ sec.

Notice, at $f_0 = 10$ Hz, RC would have to be 25 seconds for 100 degrees of freedom. However, the total "n" available from the tape loop at 10 Hz is given by $n = 2BT_r$ (Equation 55) and since $B = pf_0$, $T_r =$ record length of 10 seconds, the maximum "n" available is $n = 2pf_0T_r = (2)(.1)(10)(10) = 20$ degrees of freedom, we will achieve 100 degrees of freedom only when:

$$f_0 = \frac{n}{2pT_r} = \frac{100}{(2)(.1)(10)} = 50 \text{ Hz}$$

The total analysis time is:

$$A_t = \frac{4k}{p} \left[\frac{1}{f_1} - \frac{1}{f_2} \right]$$

$$A_t = \frac{(4)(250)}{0.1} \left[\frac{1}{10} - \frac{1}{2000} \right] = 10,000$$

$$\left[0.1 - .0005 \right] = 995 \text{ seconds}$$

or 16.6 minutes.

However, notice what happens as we let the upper frequency, f_2 , approach infinity, then $\frac{1}{f_2} \rightarrow 0$ and the analysis time approaches a limit determined by f_1 , k and p .

Thus, if the analysis frequency f_1 were allowed to start at 100 Hz, then under the previous conditions we could analyze from 100 Hz to infinity in 100 seconds, and for a tracking true averaging circuit it would only require 50 seconds.

Comparison of Analysis Time for Swept Filter Analysis

Let us now make a direct comparison of the constant percentage bandwidth analyzer with the constant bandwidth analyzer for a sample set of conditions:

Let:

1. "On-line" analysis
 2. Analysis frequency range be 50 Hz to 10,000 Hz
 3. The minimum structural bandwidth expected is 40 Hz at center frequency of 400 Hz;
- $$Q = \frac{400}{40} = 10$$
4. Analysis bandwidth of 10 Hz is required for a constant bandwidth analyzer (from Equation 59), and a 2.5% bandwidth for the constant percentage bandwidth analyzer (from Equation 59a) or $p = 0.025$
 5. Desire minimum $n = 100$
 6. Use RC averaging circuits

For the constant bandwidth analyzer:

$$n = 4BRC, RC = \frac{100}{40} = 2.5 \text{ seconds}$$

$$AR = \frac{B}{4RC} = \frac{10}{10} = 1 \text{ Hz/second}$$

Total analysis time:

$$A_t = \frac{f_2 - f_1}{AR} = \frac{9,950}{1}$$

= 9950 seconds or 2 hours and 46 minutes

For the constant percentage analyzer using fixed averaging,

$$n = 4pf_0RC, \text{ and since "n" (minimum) will occur at}$$

$$f_0 = 50 \text{ Hz}$$

Then

$$RC = \frac{n}{4pf_0} = \frac{100}{(4)(0.025)(50)} = 20 \text{ seconds}$$

$$\text{total analysis time, } A_t = \frac{4RC}{p} \ln \frac{f_2}{f_1}$$

$$A_t = \frac{(4)(20)}{0.025} \ln \frac{10,000}{50} = 3200 \ln 200 = 3200 (5.3)$$

$$A_t = 16,960 \text{ seconds or 4 hours and 42 minutes}$$

For the constant percent analyzer with a tracking RC averaging circuit

$$n = 4pk, k = \frac{100}{(.025)(4)} = 1000$$

Since $RC = \frac{k}{f_0}$, then we would adjust our tracking RC circuit for an initial time constant of $\frac{1000}{50} = 20$ seconds at 50 Hz.

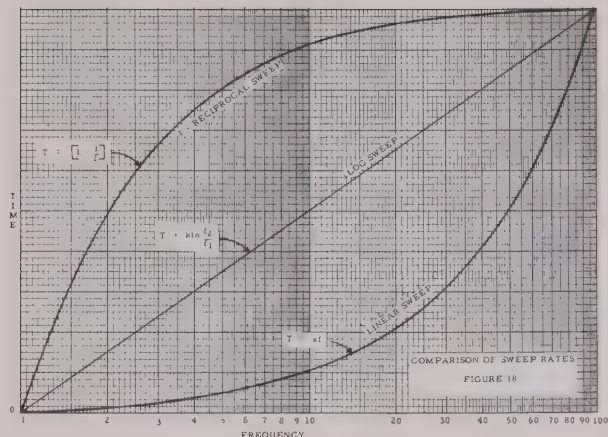
$$\text{total analysis time, } A_t = \frac{4k}{p} \left[\frac{1}{f_1} - \frac{1}{f_2} \right]$$

$$A_t = 160,000 \left[.0199 \right] = 3200 \text{ seconds or 53.3 minutes}$$

Summarizing the Total Analysis Time for the Example

For the constant bandwidth analyzer, using a 10 Hz filter, $A_t = 2$ hours, 46 minutes. For the fixed RC and constant 2.5% bandwidth analyzer, $A_t = 4$ hrs., 42 min. For the tracking RC averager and a 2.5% bandwidth analyzer $A_t = 53.3$ minutes.

A rather striking improvement! Figure 18 illustrates the relationship between the linear, log and $\left[1 - \frac{1}{f} \right]$ sweep rates. Setting the sweep rates for the above types of analyzers will be discussed next.



For the constant bandwidth analyzer, a linear sweep oscillator should be used and a setting of the analysis rate in Hz/sec. can usually be made directly with the oscillator sweep rate control.

For the constant percentage bandwidth analyzer with fixed RC averaging, the sweep rate is logarithmic with respect to time (a constant time per decade) and the sweep setting of these types of oscillators is usually in decades per minute (a decade is a ten to one frequency ratio or "n" decades = $\log_{10} \frac{f_2}{f_1}$)

Since the total analysis time for the fixed RC averaging, constant percent analyzer is $A_t = \frac{4RC}{p} \ln f_2/f_1$

and one decade is $\frac{f_2}{f_1} = 10$

Then the total time per decade is

$$A_t / \text{decade} = \frac{4RC}{p} \ln 10 = \frac{9.2RC}{p} \text{ seconds/decade}$$

therefore the log sweep rate in decades/minute

$$= \frac{60p}{9.2RC} = \frac{6.52p}{RC} \text{ decades/minute}$$

The analysis rate would be twice as fast for true averaging or AR in

$$\frac{\text{decades}}{\text{minute}} = \frac{13p}{T}$$

Where:

$$T = \text{true averaging time}$$

Next, the sweep rate for a constant percentage bandwidth analyzer using a tracking RC averaging circuit is a function of the square of center frequency of the analyzer; therefore, we would have an ever decreasing time to sweep between frequencies. We will call this type of sweep a one minus reciprocal sweep since Equation 78 can be reduced to

$$A_t \propto \left[1 - \frac{1}{f_2} \right] \text{ Where: } f_2 \geq f_1 > 0,$$

$$\frac{1}{f_1} = 1$$

$$\text{and } C = \frac{4k}{pf_1}$$

To develop this type of sweep would require the development of a special sweep oscillator or the development of a special one minus reciprocal sweep programmer circuit for an already existing sweep oscillator to be described later. Note that $\frac{1}{f_2}$ is the maximum analysis time possible.

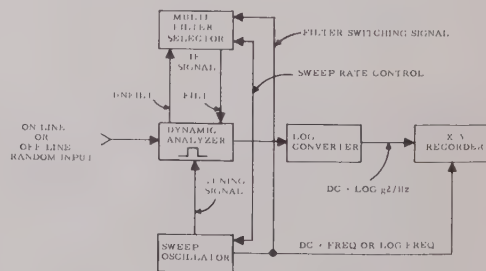
PRACTICAL ASPECTS

Since the constant percentage analyzer with steep filter skirts and a tracking averaging circuit are not available and since constant percent resolution is not desirable for all types of analysis, it would seem that an extension of the capabilities of the constant bandwidth type of analyzer could and should be made to provide a reasonable compromise.

This compromise arrangement would be to provide a constant bandwidth analyzer with a selection of filters that could be automatically switched into use over pre-programmed frequency ranges, and in conjunction with each filter provide the following: (1) bandwidth division, (2) adjustable averaging times, (3) adjustable linear scan rates or, if desired, adjustable stepped scan rates. (A stepped scan means that the analysis filter is stepped in one-bandwidth increments over the analysis frequency range.) This technique can be useful if it is desired to average over the complete length of a tape loop record and then step the analyzer one bandwidth by a command at the end of each tape loop cycle.

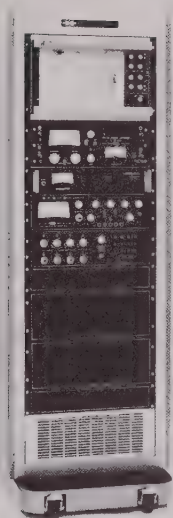
Power Spectral Density Analysis System

Such a system has been developed by Spectral Dynamics Corporation and is the Model SD1001-2 Power Spectral Density Analysis System. A system block diagram is shown in Figure 19, and a picture of the system is shown in Figure 20.



MODEL SD1001-2
POWER SPECTRAL DENSITY
ANALYSIS SYSTEM

FIGURE 19



directly in decibels. By using a logarithmic Y axis scale the power density level in g^2/Hz can be read directly off the chart.

The SD27 Multi-Filter Selector plugs directly into one of the two filter sockets in the SD101A analyzer. Thus, by using the four filters in the SD27 plus one filter in the SD101A analyzer, a total of five filters can be pre-programmed for an analysis. The standard filter bandwidths available for use in the system are 1.5 Hz, 2 Hz, 5 Hz, 10 Hz, 20 Hz, 50 Hz, 100 Hz, and 200 Hz. With this selection, a wide range of resolution is available.

RC averaging was used primarily to provide a continuous plot of the power spectrum, allowing more precise location of resonant peaks in the spectrum as opposed to the true averaging circuit, which would give an incremental display.

Sample Analysis

Let us now examine how we might set up an analysis program using the features of the SD1001-2 system. Assume the following:

1. Tape loop length = 10 seconds = T_r
2. Analysis frequency range = 10 Hz to 2000 Hz
3. n desired = 100 degrees of freedom
4. Max. structural Q expected = 5

We must first select the filters that will provide proper resolution of the spectrum.

Referring back to Equation 61

$$Q_a = 4Q \text{ (max)} = 20$$

to compute the percent resolution,

$$\text{Since } Q_a = \frac{f_0}{B} \text{ (using Equation 62) and (using Equation 59 a) resolution} = \frac{1}{Q_a} \times 100 = \frac{1}{20} \times 100 =$$

5% = minimum resolution to meet the criteria given in Equation 61.

The next step is to select the switching frequency so that the minimum resolution will not be exceeded. Since the lowest frequency is 10 Hz, the bandwidth that will provide a minimum resolution can be computed using % resolution = $\frac{B}{f_0} \times 100$

$$\text{therefore: } B = \frac{\%R \times f_0}{100} = \frac{(5)(10)}{100} = .5 \text{ Hz}$$

The system is comprised of the following:

1. An SD101A Dynamic Analyzer which is a tracking, heterodyne, constant bandwidth analyzer. It provides automatic band-switching between two analysis bandwidths and uses a linear absolute average detector.
2. An SD104A linear, logarithmic and programmable Sweep Oscillator with front panel and remote sweep rate control; log and linear dc analogs proportional to the oscillator frequency.
3. An SD27 Multi-Filter Selector, providing four automatically or manually switched filter bandwidths, and providing with each filter (a) an adjustable RC averaging circuit, (b) an analysis sweep rate control for remote control of the SD104A sweep rate, (c) a filter phase shift normalization control, (d) a bandwidth division control, (e) a switching frequency control, and (f) indicator lights showing the selected filter.
4. A log converter for squaring the averaged absolute value from the SD101A analyzer; thus providing the ability to plot a wide dynamic range of power density estimates.
5. An X-Y recorder operated by the output of the log converter and the desired frequency analog from the sweep oscillator, thus providing automatic plots of the logarithm of the power spectral density vs. frequency (linear or log frequency scale). By using a linear Y axis scale, a plot of the power spectral density can be read

Since we only have a 1.5 Hz filter, we will not be able to meet the criteria of $\frac{B_8(\min)}{4}$. Remembering that the % resolution of a constant bandwidth analyzer decreases with frequency, we will not be able to meet the criteria of 5% resolution until we reach an analysis frequency of $\frac{(100)(1.5)}{5} = 30$ Hz (from Equation 59a).

We will be able to compute the crossover frequencies for the remainder of the filters from the equation

$$f_0 = \frac{B}{k} \quad \text{where: } B = \text{filter bandwidth} \\ f_0 = \text{analysis center frequency} \\ k = \frac{\% \text{ resolution}}{100}$$

Compute minimum resolution frequency for filter bandwidths available.

B = 1.5, 2, 5, 10, 20, 50, 100, 200

$$(1) f_0 = \frac{1.5}{.05} = 30; (2) f_0 = \frac{2}{.05} = 40;$$

$$(3) f_0 = \frac{5}{.05} = 100; (4) f_0 = \frac{10}{.05} = 200;$$

$$(5) f_0 = \frac{20}{.05} = 400; (6) f_0 = \frac{50}{.05} = 1000;$$

$$(7) \frac{100}{.05} = 2000; (8) f_0 = \frac{200}{.05} = 4000$$

From the above, we can rule out the 100 Hz and 200 Hz bandwidth filter since they can't meet the minimum resolution within the analysis frequency range. Therefore, we have a choice of the remaining six. Without prior knowledge of the spectrum, we would have to choose the 1.5 Hz filter to try and properly resolve the spectrum. Therefore, we can use the 1.5 Hz filter from 10 Hz to 40 Hz, the 5 Hz filter from 40 Hz to 200 Hz, the 10 Hz filter from 200 Hz to 400 Hz, the 20 Hz filter from 400 Hz to 1000 Hz and the 50 Hz filter from 1000 Hz to 2000 Hz. We sacrificed some resolution by not using the 2 Hz filter, however, we will improve the statistical confidence and the analysis time by using the 5 Hz filter.

Averaging times: $n = 4BRC$, $n_{\max} = 2BT_r$

(1) For the 1.5 Hz filter; $n_{\max} = (2)(1.5)(10) = 30$ degrees of freedom

$$RC = \frac{n}{4B} = \frac{30}{6} = 5 \text{ seconds}$$

(2) For the 5 Hz filter; $n_{\max} = (2)(5)(10) = 100$ degrees of freedom

$$RC = \frac{n}{4B} = \frac{100}{20} = 5 \text{ seconds}$$

(3) For the 10 Hz filter; $RC = \frac{100}{40} = 2.5 \text{ sec.}$

(4) For the 20 Hz filter; $RC = \frac{100}{80} = 1.25 \text{ sec.}$

(5) For the 50 Hz filter; $RC = \frac{100}{200} = 0.5 \text{ sec.}$

$$AR = \frac{B}{4RC}, \text{ therefore } AR_1 = \frac{1.5}{20} = 0.075 \text{ Hz/sec.}$$

$$AR_2 = \frac{5}{20} = 0.25 \text{ Hz/sec.}$$

$$AR_3 = \frac{10}{10} = 1 \text{ Hz/sec.}$$

$$AR_4 = \frac{20}{5} = 4 \text{ Hz/sec.}$$

$$AR_5 = \frac{50}{2} = 25 \text{ Hz/sec.}$$

Total analysis time will be the sum of the time for analysis for each filter or:

$$A_t = \left[\frac{f_2 - f_1}{AR_1} \right] + \left[\frac{f_3 - f_2}{AR_2} \right] + \left[\frac{f_4 - f_3}{AR_3} \right] + \left[\frac{f_5 - f_4}{AR_4} \right] + \left[\frac{f_6 - f_5}{AR_5} \right] \quad (79)$$

f_1 is the lowest analysis frequency, f_6 is the highest analysis frequency f_2 through f_5 are the programmed switchover frequencies of the SD27.

From our example:

$$A_t = \left[\frac{40 - 10}{.075} \right] + \left[\frac{200 - 40}{.25} \right] + \left[\frac{400 - 200}{1} \right] +$$

$$\left[\frac{1000 - 400}{4} \right] + \left[\frac{2000 - 1000}{25} \right]$$

$$A_t = 1430 \text{ seconds or } \approx 24 \text{ minutes}$$

Had we had some prior knowledge about the spectrum, we could have optimized the bandwidths and probably saved additional time.

As was stated earlier, there is always a certain dilemma in making the compromise between resolution, statistical accuracy, and analysis time. However, from the foregoing discussions the ability to automatically program the filter bandwidths of a constant bandwidth analyzer along with the other associated parameters required, allows a greater overall optimization of the resolution, analysis time and statistical accuracy of power spectral density analysis. In turn, we have still maintained the flexibility of a constant bandwidth analyzer for the analysis of periodic, complex and transient signals.

CONCLUSIONS

The analysis of periodic, complex, transient, certain types of quasi-sinusoids and random signals must be accomplished using a constant bandwidth analyzer to obtain proper resolution of the spectrum. The analysis of random signals from real structures could be better analyzed using a constant percentage bandwidth analyzer with a frequency tracking averaging circuit; however, the filter characteristics of present day constant percentage bandwidth analyzers and the lack of flexibility make these type analyzers generally unacceptable for power spectral density analysis.

The use of a linear detector on the output of a power density analyzer can be justified due to cost, simplicity, and continuous dynamic range.

The averaged output from a linear detector provides an adequate measure of the power density if the filtered signal has a Gaussian amplitude distribution. Due to the central limit theorem, departures from a Gaussian distribution are very slight when narrow band non-Gaussian noise is filtered by an analysis filter of the same noise bandwidth. RC averaging of the detected analyzer output is generally preferred since it provides a continuous spectrum display.

The addition of programmed filter bandwidth switching along with the associated parameters of averaging time, analysis scan rate and bandwidth division to the constant bandwidth analyzer offers a reasonable compromise approach to reducing analysis time, obtaining good statistical accuracy with adequate resolution, while retaining the flexibility of the constant bandwidth analyzer. This type of a system has been developed and is the Spectral Dynamics SD1001-2 Power Spectral Density Analysis System.

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APPENDIX I

Empirical Data on the Effects of Filtering Non-Gaussian Random Noise

The purpose of obtaining this data was to observe the effects of filtering various bandwidths of non-Gaussian random noise through various wave analyzer bandwidths.

It has been stated by many people involved in the analysis of random data that narrow band filtering of signals with non-Gaussian probability density tends to make the probability density of the filtered output Gaussian, provided the filter bandwidth is small compared to the bandwidth of the signal being analyzed.

This investigation was to provide an indication of the probability densities when the non-Gaussian noise bandwidth was close to that of the analysis filter.

Test Equipment

The test equipment used comprised the following:

1. Two Spectral Dynamics Model SD101A Dynamic Analyzers employing crystal lattice filters with shape factors of 4.
2. An H. H. Scott, Type 811-B Audio Frequency Random Noise Generator
3. A Ballantine Laboratories Model 320A, True RMS Voltmeter
4. A Gulton, Ortholog Division, Probability Density Analyzer
5. A Krohn-Hite Bandpass Filter, Model 330M, .2 Hz - 20 kHz
6. A Hewlett-Packard, Model 3440A, dc digital voltmeter with a 3443A Plug-In.
7. Moseley, Model 135, X-Y Recorder

Several special considerations were maintained throughout the test. (a) Since the source of the random data was virtually unlimited in time, we were able to average the signals from the PDA (Probability Density Analyzer) for a period long enough to maintain a statistical error of approximately 5% above a probability density of .04. Below this a WBT product was maintained to provide an error of approximately 10%.

From the expression -

$$\sigma_{\%} \epsilon = \frac{0.2}{\sqrt{p(y)} \text{ WBT}}$$

Where $p(y)$ is the measured probability density and W is the amplitude of the gate width in relative amplitude = .1 for the analyzer used.

T is the averaging time.

The final readout was made, point by point, using an X-Y recorder. The final graphs presented were obtained by transferring the mean reading of the recording at each point to the final graph. Prior to plotting each data point the gate width and position of the PDA was calibrated using the digital voltmeter.

Test Program

A noise spectrum of 10 Hz to 20 kHz was established by band limiting the random noise generator with a Khron-Hite bandpass filter. The output of the Khron-Hite was then considered the random noise source for some analyses; for others, the noise source was filtered by a wave analyzer. The wave analyzer output was then considered the noise source. In the figures the circled letters show the point in the circuit block diagram where the PDA was made.

The method of creating the random signal with non-Gaussian amplitude probability density was by the use of a germanium diode connected to clip the negative signals from the noise source at around 250 millivolts.

In all cases involving the use of the wave analyzer outputs, the analysis filter was tuned to a center frequency of 1000 Hz.

Probability Density Analysis Program

- | | |
|---|-----------|
| 1. PDA of wideband random source | Figure 1X |
| 2. PDA of 100 Hz bandwidth filter with broadband input | Figure 2X |
| 3. PDA of 20 Hz bandwidth filter with broadband input | Figure 3X |
| 4. PDA of 5 Hz bandwidth filter with broadband input | Figure 4X |
| 5. PDA of negative amplitude limited broadband input | Figure 5X |
| 6. PDA of 100 Hz filter with negative amplitude limited broadband noise input | Figure 6X |
| 7. PDA of 20 Hz bandwidth of negatively limited noise | Figure 7X |
| 8. PDA of 20 Hz filter with a 20 Hz bandwidth of negative amplitude limited noise input | Figure 8X |
| 9. PDA of 20 Hz filter with 100 Hz bandwidth of negative amplitude limited noise input | Figure 9X |

10. PDA of 100 Hz filter with 20 Hz bandwidth of negatively limited noise input
11. Obtain sample waveforms of:
 - (a) Broadband clipped random
 - (b) Broadband random signal
 - (c) Clipped 5 Hz bandwidth of noise
 - (d) 5 Hz bandwidth of random noise
 - (e) Clipped 100 Hz bandwidth of noise
 - (f) 100 Hz bandwidth of random noise

Figure 10X

Figure 11Xa

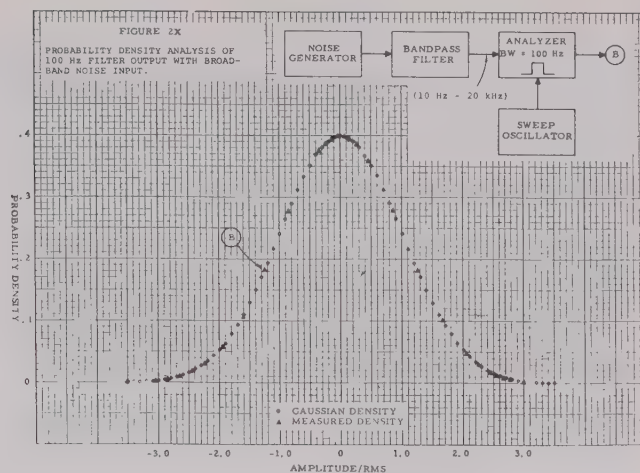
Figure 11Xb

Figure 11Xc

Figure 11Xd

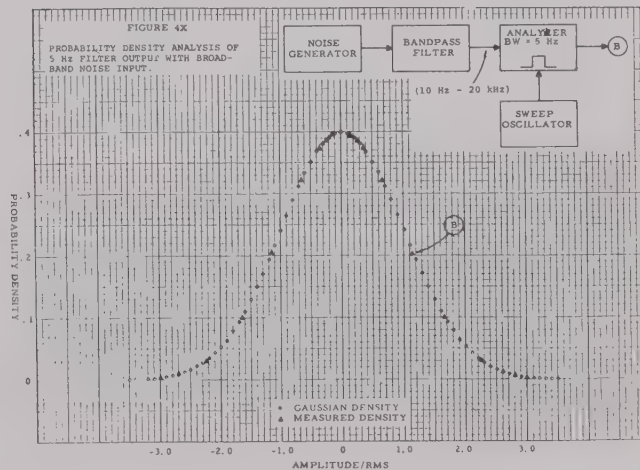
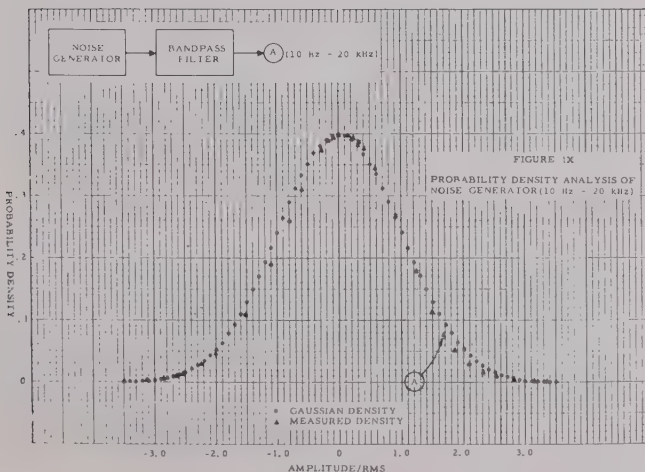
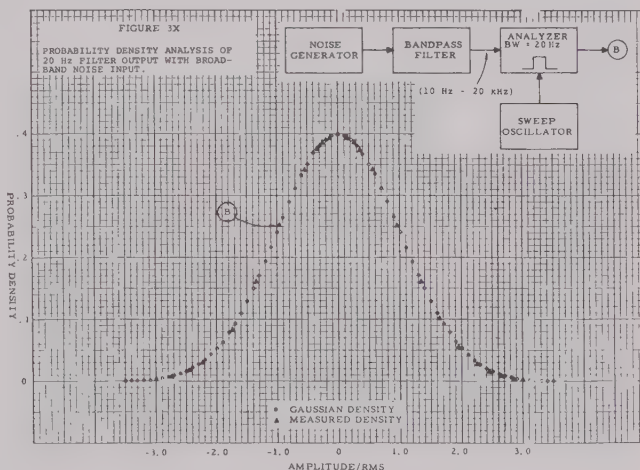
Figure 11Xe

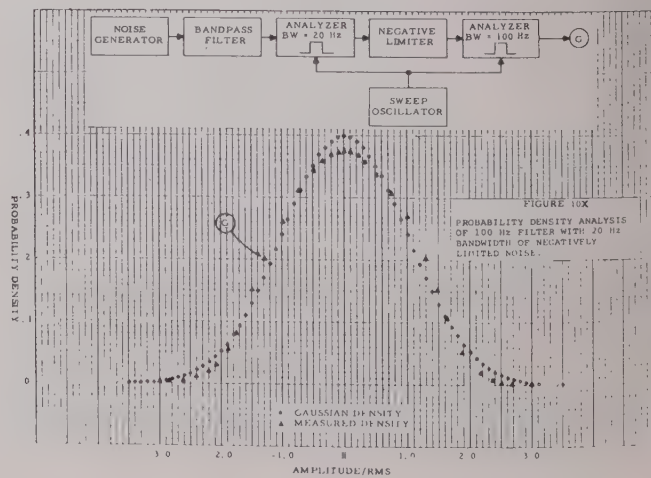
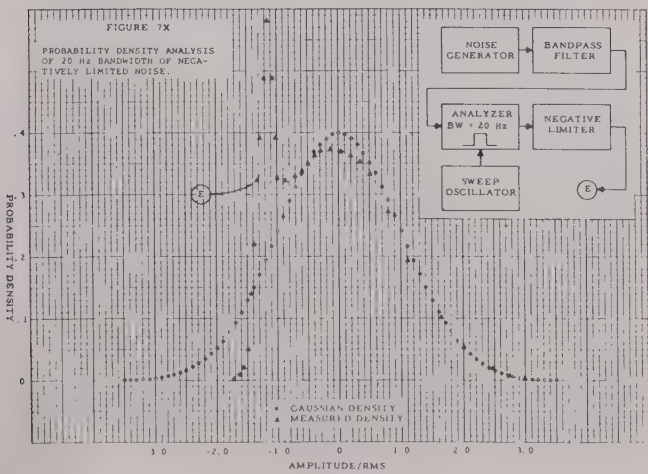
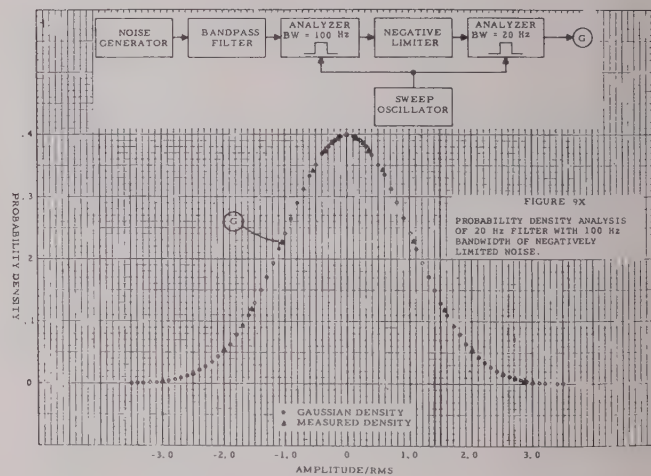
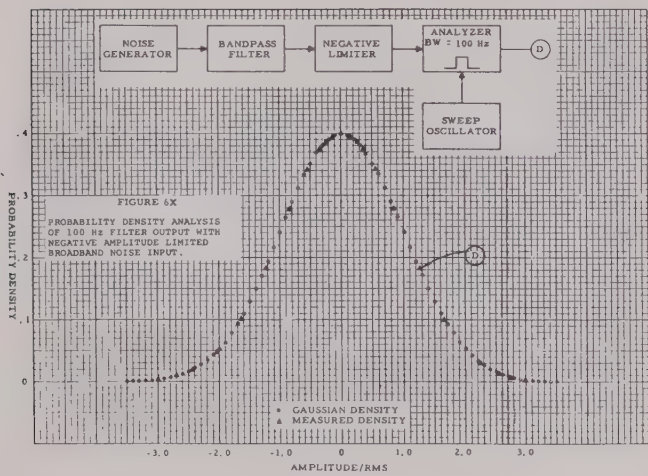
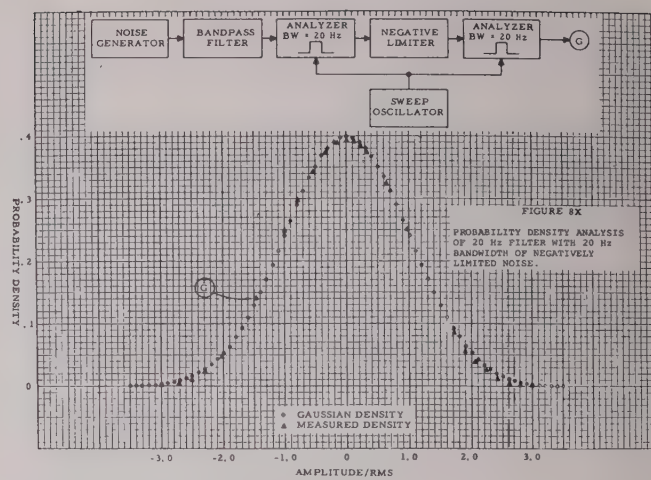
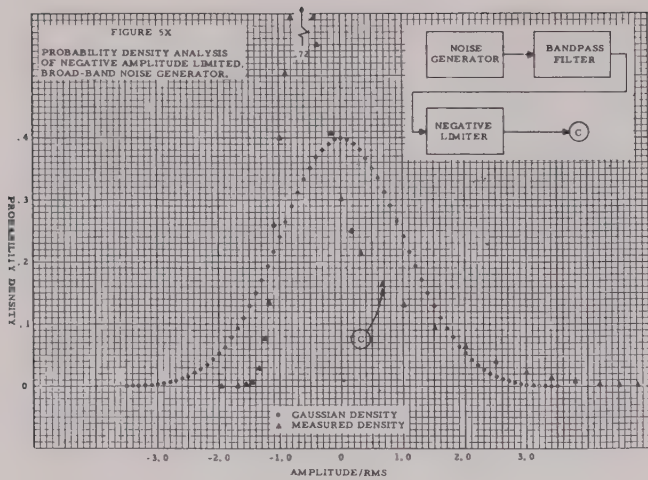
Figure 11Xf



Discussion of Results & Conclusions

A review of the data indicates that there was very little departure of the signals at the output of the analysis filter from a Gaussian Probability Density with a non-Gaussian noise bandwidth as narrow as to be equal to the analysis filter bandwidth. When the analysis bandwidth was five times that of the non-Gaussian band of noise, there was a noticeable departure from Gaussian probability density. An analysis of Figure 10X shows that for this "distorted" case, the absolute average value is 0.816 times the rms value. This represents an error of approximately 2.5% with respect to the true Gaussian case.





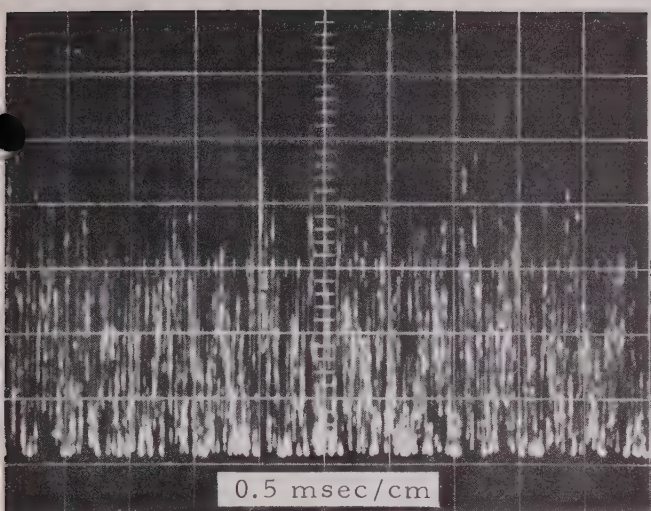


FIG 11a. RANDOM CLIPPED

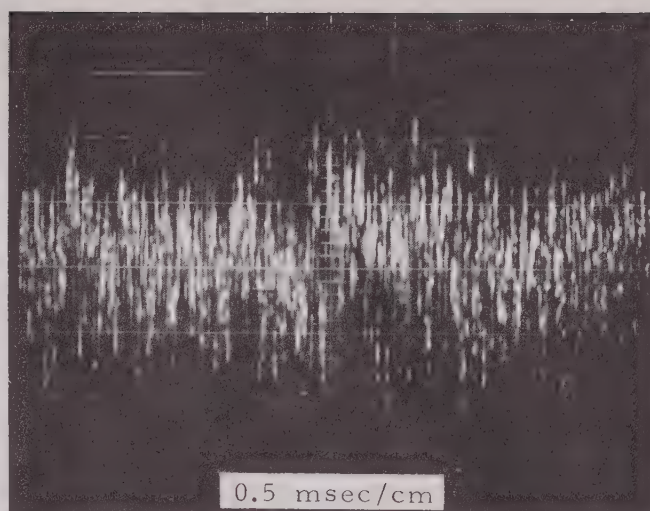


FIG 11b. RANDOM

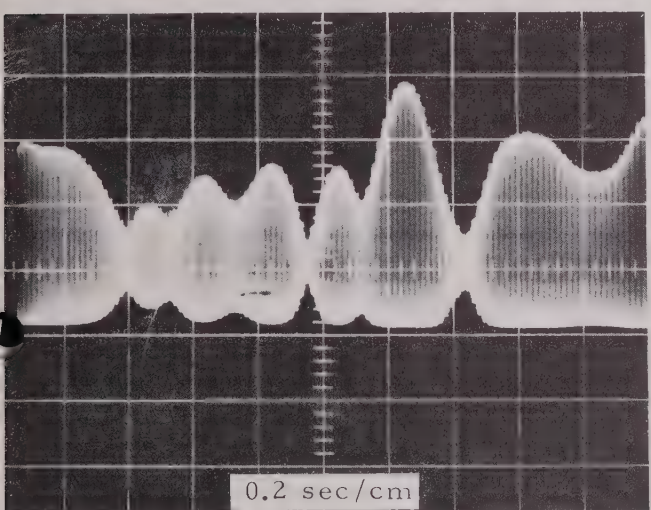


FIG 11c. FSO CLIPPED, 5 CPS FILTER
TUNING SIGNAL = 2 KCPS

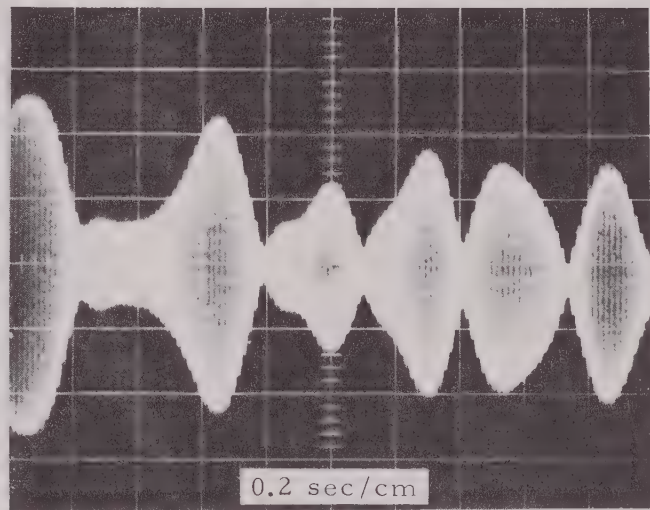


FIG 11d. FSO, 5CPS FILTER
TUNING SIGNAL = 2 KCPS

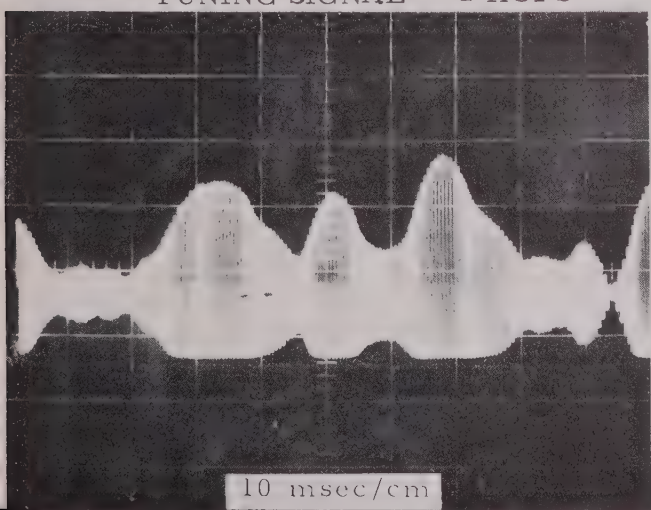


FIG 11e. FSO CLIPPED, 100 CPS FILTER
TUNING SIGNAL = 2 KCPS

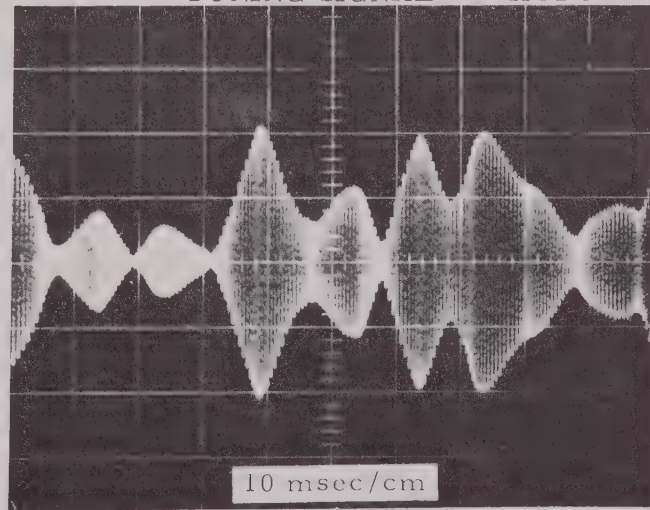


FIG 11f. FSO, 100 CPS FILTER
TUNING SIGNAL = 2 KCPS

FIGURE 11X- TYPICAL RANDOM SIGNALS

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Considerations In The Analysis Of Arbitrary Waveforms

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CONSIDERATIONS IN THE ANALYSIS OF ARBITRARY WAVEFORMS

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INTRODUCTION

All signals can be thought of as existing in three dimensions. As shown in Figure 1, a sinewave is in reality an amplitude-time curve existing at some fixed point on the frequency axis. An examination or projection of this sinusoidal time history on the amplitude-frequency plane would be a vertical line at the frequency f_1 . This is the true spectrum of a sinewave and can be viewed as the two-dimensional projection of a time function onto the amplitude-frequency plane. Such a projection of the frequency parameters of a time-amplitude waveform is called a spectrum or frequency analysis.

Similarly the properties of this amplitude-time waveform could be viewed in the amplitude-time domain. One immediate result would be the original sinusoidal waveform. If, instead of viewing the waveform in real time, a pseudo time equal to the time difference between sampled points on the waveform were used, a correlation function would result.

If the arbitrary time function under investigation were viewed in the amplitude-time domain, not with respect to a time difference between points on the function, but with respect to the percentage of time that the function stays within certain amplitude limits, a probability function would result. This again would be a function of amplitude and time and would be truly independent of frequency.

We have, then, three situations which immediately come to light:

In the first case, a signal is investigated for its frequency content. This is done by averaging the signal over a certain time interval and using the appropriate Fourier weighting function. The result of this transformation is a function in the frequency-amplitude domain from which time has essentially been averaged out.

In the second case, an average value is again obtained through integration with respect to time,

resulting in a function which is described in the pseudo domain of time difference, tau (τ).

In the third case, the arbitrary time function is operated on in an amplitude and time dependent sense, with a result that is completely independent of frequency but is averaged over time and is in reality plotted on an amplitude-amplitude basis.

Each of the three functions, frequency spectrum, time correlation and amplitude probability, have important engineering significance.

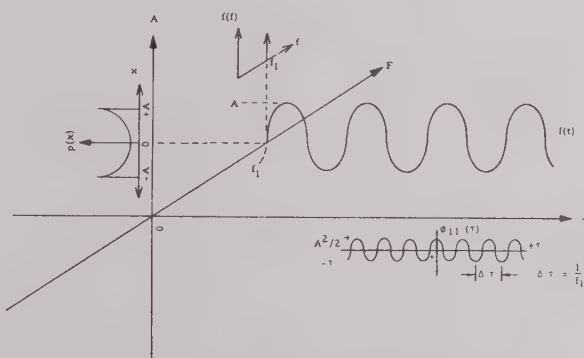


FIGURE 1. THREE-DIMENSIONAL REPRESENTATION OF AN ARBITRARY WAVEFORM.

CORRELATION

Correlation techniques have been used for many years to extract a signal from noise or to reveal the presence of a periodicity in an otherwise completely random signal. Thus, if a signal is correlated with itself (autocorrelation), the random component of the signal will reduce rapidly to zero in the correlation domain, depending on the bandwidth of the noise present, and any periodic component will immediately be revealed as a continuing function in the time delay domain. This can be shown for non-periodic signals as an autocorrelation function $\phi_{11}(\tau)$:

$$(1) \quad \phi_{11}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f_1(t) f_1(t + \tau) dt$$

Similarly, if two signals are crosscorrelated, one with another, an immediate result is the indication of any and all mutual properties which these signals may share. Thus, a crosscorrelation function $\phi_{12}(\tau)$ is defined

$$(2) \quad \phi_{12}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f_1(t) f_2(t + \tau) dt$$

where T = time over which observation and integration are performed. One important application of crosscorrelation techniques is the determination of which of many inputs are related to a specific output. If a force is measured at one point on a rocket vehicle and it is necessary to determine which, if any, possible input most directly caused the resultant measured force, a crosscorrelation analysis between the resultant force and each of the possible inputs will reveal which input is most closely related to the measured output force.

PROBABILITY

Probability functions have been extremely useful for many years in the prediction of the behavior and structural response of many tactical vehicles. For example, in the early days of random vibration testing, although it had been shown that the output of random noise generators was truly Gaussian, very little work had been done on the actual estimation or measurement of the probability function associated with in-flight random vibration. Required stress levels were often simulated in the laboratory, based on only a knowledge of the overall rms vibration and the assumption that the probability density function which the data followed was truly Gaussian. In the late 1950's, economic, automatic probability density analyzers first became available. When these analyzers were applied to the analysis of in-flight vibration data, surprisingly perhaps, much of the data was found to truly approach the Gaussian distribution. However, other data which had been readily assumed to follow a Gaussian distribution was seen to be in gross error.

By the same token, many of the tests for equivalence between sinusoidal and random vibration qualification testing have depended to a large extent on the assumption that the random vibration environment seen by a specimen during actual usage indeed followed a Gaussian density function. Once again, this was in many cases a valid assumption; however, the availability now of automatic probability density analyzers and event counters enable the design of qualification and equivalent tests to be approached from a more scientific standpoint.

A probability density function can be defined as:

$$(3) \quad p(y) = \lim_{T \rightarrow \infty} \frac{1}{T(\Delta y)} \sum_{i=1}^{\infty} t_i(y, y + \Delta y) \Delta y \rightarrow 0$$

Where t_i is the amount of time the signal stays in an amplitude window Δy units wide.

SPECTRUM ANALYSIS - WHICH FUNCTION?

Although both probability and correlation functions are seen to be extremely useful engineering tools, by far the most widely used and generally understood signal presentation has been a plot of spectral content. Several factors have contributed substantially to this; among these have been the long history of Fourier analysis, the ready availability of wave analyzers for many years, the acceptable signal-to-noise enhancement possible with commercially available analyzers and the close relationship between signal spectrum content and general test specifications. (An additional factor has historically been the difficulty of physically making accurate correlation measurements over wide ranges of amplitude and time delay.)

Once it has been decided that the properties of an arbitrary signal are best described for a specific case through the use of its spectral content, the first question which arises is exactly which amplitude parameter in the frequency domain shall be plotted. If the possibilities are examined, there results a choice between peak values, average values, rectified average values, effective root mean square values or higher order (power) values.

Since acceleration waveforms are often recorded, viewed and compared based on the output of a crystal accelerometer, which is a first power transducer, a first power parameter would seem most amenable to any comparison. It is obviously impractical to speak of the comparison of peak values, since for a random acceleration waveform, the peak acceleration in theory can approach infinity. Likewise, the average acceleration of most stable vehicles is, in most cases, zero. Thus, a choice boils down to that between rectified average values and true root mean square or effective values.

If the probabilities and correlation functions mentioned earlier are examined in a side-by-side comparison with a spectrum function, it is seen that the standard deviation associated with a probability function and the correlation coefficient associated with zero time delay are both related to the effective or true root mean square value of the waveform under analysis. This is shown in Figure 2. In addition, many voltmeters are calibrated to read the true rms value of sinusoidal calibration signals and for years a root mean square value has been the term accepted by hosts of electrical engineers. For these reasons in addition to the close correlation which then results from spectrum, probability and

correlation analysis, the use of a spectrum parameter from which rms values may be determined has long been accepted in the vibration field.

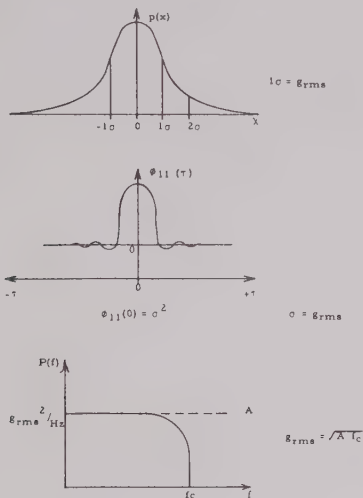


FIGURE 2. EXAMPLE ANALYSIS OF FILTERED RANDOM ACCELERATION WAVEFORM

Once it has been determined that a final answer in terms of rms acceleration must be achieved a hard look must next be taken at the techniques to be employed in performing a spectrum analysis. Under most conditions a single reading of total wide band root mean square acceleration is not sufficient. If this were sufficient a simple analysis could be quickly made using only a broadband rms meter. However, information must be obtained which relates both to the spectral content of the waveform under analysis and to its effective root mean square value over an arbitrary portion of the total spectrum. Thus, if the root mean square acceleration between two frequencies f_1 and f_2 is available to a design engineer, he has a ready means of comparison between resultant data, test specifications and additional types of analyses such as correlation or probability density which may have been performed.

If one were to examine a spectrum analysis presented in terms of root mean square acceleration versus frequency, an immediate problem would come to light. Although it would be quite straightforward to read off an rms value of acceleration at a specific frequency the determination of root mean square acceleration over a frequency band would be extremely difficult. This is in essence so since voltages and accelerations may not be combined, in general, as a linear addition, but rather must be combined as the square root of the sum of the square of individual components.

As a general rule the only voltages which may be added directly are dc voltages or those vector quantities which are exactly in phase. Since neither of these represents a practical acceleration

waveform a frequency spectrum performed on the basis of g_{rms} vs. frequency would not seem to be the answer. Rather a term proportional to a squared voltage, or power, would be more convenient if our intermediate goal is the addition of values over a certain portion of the spectrum. Thus, use of a power term will facilitate the addition of spectrum values between different arbitrary frequencies since direct addition or summation of values is then valid.

However, one additional point remains to be satisfied. This is, that if spectrum values are to be summed between two specific frequencies to achieve a resultant overall level, the final answer depends critically upon the absolute magnitude of the quantities being added. It is known that for a random input the output of a bandpass filter increases with filter bandwidth; thus, if two spectrum analyses were performed on a common data sample each with different bandwidths, the analysis performed with the wider bandwidth would result in a higher apparent overall rms acceleration. This would obviously not be a true result. Naturally it is often desirable to perform analyses with different bandwidths since finer resolution may be obtained by using a narrower filter. However, the process of providing better resolution should be compensated in the analysis to the extent that it does not at the same time provide an erroneous answer. Since for a white noise input the power output of a bandpass filter increases directly proportional to effective bandwidth (thus, the voltage output of a bandpass filter increases proportional to the square root of bandwidth increase) an obvious solution is to eliminate the effect of filter bandwidth on the analysis performed. This is most easily accomplished by dividing the total squared analysis by the effective squared bandwidth used.

We have thus seen that in order to provide a useful quantity which can be related both to other analyses and to test requirements and at the same time arrive at our required data in a simple straightforward manner from a graphical analysis, yet make the analysis amenable to comparison with analyses performed using different filters, a graphical presentation in terms of mean squared acceleration divided by filter bandwidth is in one sense an optimum choice. This term is usually referred to as Power Spectral Density (g_{rms}^2 / Hz).

DEFINITION OF PSD

The Power Spectral Density of a signal is independent of the phase of the signal with respect to some arbitrary reference. Thus two signals identical in nature but differing only in relative phase will have identical power spectra. Mathematically, Power Spectral Density is related to the Fourier transform of a signal. Thus, if we define a Fourier transform, $F(\omega)$, such that

$$(4) \quad F(\omega) = 1/2 \pi \int_{-T/2}^{T/2} f(t) e^{-j\omega t} dt$$

then a power spectral density function can be arrived at as a normalized product between a Fourier transform and a complex conjugate transform as shown in equation 5.

$$(5) \quad P(\omega) = \lim_{T \rightarrow \infty} \frac{1}{T} [F(\omega) \cdot F^*(-\omega)]$$

A more satisfying derivation of Power Spectral Density (Ref. 1) is available based on Parseval's theorem. Parseval's theorem, in general, states that the total energy associated with a waveform in the time domain is equal to the energy of the same waveform when translated to the frequency domain. Thus, the average power for a periodic function can be stated as:

$$(6) \quad P_{ave} = \frac{1}{2T} \int_{-T}^T [f(t)]^2 dt$$

For a non-periodic function, $f_n(t)$, this becomes

$$(7) \quad P_{ave} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [f_n(t)]^2 dt$$

where $f_n(t)$ should be continuous such that

$$\int_{-T}^T [f_n(t)]^2 dt \text{ is finite}$$

and

$$\int_{-\infty}^{\infty} [f_n(t)]^2 dt \text{ exists.}$$

By Parseval's theorem,

$$(8) \quad \int_{-\infty}^{\infty} [f(\beta)]^2 d\beta = \int_{-\infty}^{\infty} [F(f)]^2 df$$

where

$$(8a) \quad F(f) = \int_{-\infty}^{\infty} f(\beta) e^{-j2\pi f\beta} d\beta$$

and

$$(8b) \quad f(\beta) = \int_{-\infty}^{\infty} F(f) e^{j2\pi f\beta} df$$

are a Fourier transform pair.

Through a change of limits in equation 7 we can write

$$(9a) \quad P_{ave} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [f(t)]^2 dt$$

$$(9b) \quad = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-\infty}^{\infty} [f(t)]^2 dt$$

substituting (8) into (9)

$$(10a) \quad P_{ave} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-\infty}^{\infty} [f(t)]^2 dt$$

$$(10b) \quad = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-\infty}^{\infty} [F(f)]^2 df$$

$[F(f)]^2$ is an even function, or in other words the power in the negative side of the two sided spectrum is equal to the power in the positive half. Thus,

$$(11a) \quad P_{ave} = \lim_{T \rightarrow \infty} \frac{1}{2T} 2 \int_0^{\infty} [F(f)]^2 df$$

$$(11b) \quad = \lim_{T \rightarrow \infty} \int_0^{\infty} \frac{[F(f)]^2}{T} df$$

$$(11c) \quad = \lim_{T \rightarrow \infty} \frac{1}{T} [F_0(f)^2 + F_1(f)^2 + \dots + F^\infty(f)^2] \Delta f$$

$$(11d) \quad = \lim_{T \rightarrow \infty} \sum_{n=0}^{\infty} \frac{[F_n(f)]^2}{T} \Delta f$$

The Power Spectral Density, $P(f)$, is then

$$(12) \quad P(f) = \frac{P_{ave}}{\Delta f}$$

where Δf is the bandwidth of a realizable filter which approaches zero Hz in the limit.

PSD ANALYZERS

In commercial PSD analyzers three steps are performed. First, the signal to be analyzed is introduced to a narrow band filter which is to be swept over the frequency range of interest. Secondly, after filtering, a power function is formed from the filtered signal output. Finally, a normalized average product is formed through integration and division by effective filter bandwidth.

From the preceding derivation, we see that a function $P(f)$ has been defined such that

$$(13) \quad P(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y_B^2 dt$$

where

$P(f)$ = Power Spectral Density (PSD)

y_B^2 = instantaneous square of signal within a narrow bandwidth, B

B = signal bandwidth (Hz)

This says that the true power spectral density function is obtained by averaging the squared value of a signal in a narrow bandwidth B , over an infinite time period with an infinitesimally small filter bandwidth. In practice, we don't have infinitely long random records nor the time to average to infinity, and we don't have infinitesimally narrow bandwidths. Thus, we can never attain the true PSD.

We must simply estimate the power spectral density function from a finite length of record, averaging over a finite time interval with a finite bandwidth. Equation 13 then becomes

$$(14) \quad \tilde{P}(f) = \frac{1}{T} \int_0^T \frac{y_B^2}{B} dt = \overline{\frac{y_B^2}{B}}$$

where

$\tilde{P}(f)$ = an estimate of the power spectral density

T = a finite averaging time

B = the effective noise bandwidth of the analysis filter

$\overline{\frac{y_B^2}{B}}$ = the average of the square of the amplitudes within the analysis bandwidth

Note that equation 14 assumes a flat spectrum within the passband of the analyzer.

Once a suitable filtering technique has been selected (such as crystal filter/heterodyned, active low-pass filter/modulation etc.), the next consideration is that of performing a power function operation. From the rigorous definition of PSD, a squaring operation would seem to be in order. However, the limits of the power spectral density derived from much of the data available today indicates variations in PSD levels of as much as 80 dB. A true squaring circuit would thus be required to operate over approximately 80 dB of dynamic range. Until very recently, this was completely impossible and even at this time the design of a circuit to operate in a useful manner over this dynamic range is quite impractical. Noise level, linearity and instability are three considerations working against the practical use of wide dynamic range, multiplication circuits.

Let us now investigate some of the properties of filtered random signals to see if these properties can in some way assist us in our analysis.

EFFECT OF FILTERING ON RANDOM SIGNALS

If a broad band random signal which contains no true periodic components is analyzed, several properties become immediately apparent. The spectrum function of the broad band signal may be flat or may have some arbitrary peaks and notches in it. The probability function of the broad band signal would most likely be some arbitrary function, not necessarily Gaussian. By the same token, if the signal were truly random, its autocorrelation function would decrease rapidly to zero for both positive and negative time delay. The application of this signal to a true rms meter would result in a steady indication of the signal rms level. If an oscillograph of the wideband signal were made or if an oscilloscope presentation were carefully analyzed, it would be noted that on the positive side of the signal's mean value, both

positive and negative, going peaks occurred and likewise on the negative side of the signal's mean value, positive and negative going peaks would appear.

If the broad band signal is now processed through successively narrower band pass filters several interesting changes occur. In the first place, as the filter bandwidth becomes narrower, the rms meter indication begins to show more fluctuations. Secondly, the spectral content of the filtered signal naturally begins to compress in the frequency domain such that only frequencies within the filtered data bandwidth display significant energy. Thirdly, as the bandwidth of the filter reaches a certain critical width, the oscilloscope or oscillograph presentation no longer indicates negative going peaks on the positive side of the mean value of the signal and vice versa. The signal will in reality begin to appear as an amplitude modulated sine wave with a slight phase deviation. In the fourth place, the autocorrelation function of the filtered signal begins to spread out in the time delay domain. The time over which the signal remains essentially correlated is seen to be an exact function of the bandwidth of the filter. Finally and most important perhaps, it will be noted that as the bandwidth decreases beyond a certain point, the probability density function of the instantaneous amplitudes begin to almost perfectly follow the Gaussian density function.

It has been shown by many test laboratories that if a probability density analysis is performed, for example, on the output of an FM reproduce amplifier when a tape machine has been stopped, the resulting probability function will be in most cases quite distorted and not at all Gaussian. If, however, the noise signal out of the reproduce amplifier is filtered by a bandpass filter placed in the spectrum of the signal where significant energy exists, the probability function of the filter output will approach an almost perfect Gaussian density function. This can be of extremely important significance if one is attempting to measure the effective value of an arbitrary waveform.

If a signal of unknown statistical distribution is examined, the ratio of its rectified average to effective or root mean square value can be almost any quantity. For a sine wave, the ratio of effective to rectified average values is known to be 1.11. Similarly, for a random signal which follows a Gaussian density function it is known that the ratio of rms to average is 1.25. (See Appendix) Thus, average detecting meters, for instance, which are calibrated for use with sine waves and which give erroneous indications upon the application of a wave form other than sinusoidal can be used quite readily to measure effective values if it is known that the input signal follows the Gaussian distribution.

This information then can be used to considerable advantage by an engineer desiring to perform power spectral density estimates on random waveforms through the use of narrow band filters. Since it has

been shown that the output of a narrow bandpass filter when a wide band random signal is applied does truly approach a Gaussian density function, a linear detector with a simple correction factor applied can readily be used in lieu of the squaring or square law detector for the true estimation of power properties of the filtered signal. Thus, for example, if the output of a narrow bandpass filter is detected using an average detector and a correction factor of 1.25 is applied, a resultant quantity will be obtained which is directly proportional to the effective or root mean square value of the filtered signal output. In order to complete the PSD analysis in this fashion, analysis filter bandwidth must be removed from data consideration and a signal presented as a power function for plotting.

Since in many practical cases investigations of structural resonances involve PSD variations of greater than 40 dB, data has been most often presented on a logarithmic (ordinate) basis. This tends to make frequent rescaling unnecessary and provides equal resolution over an entire range of amplitude values.

We have seen then that through a simple calibration technique a linear detector may be applied to the output of a narrow bandpass filter to give a resultant indication in terms of true rms voltage. Also, from equation 12, it is seen that power spectral density is in essence average power divided by effective filter bandwidth. Since the square root of average power is by definition true rms voltage, the squaring of an effective (rms) value followed by bandwidth normalization will indeed result in a true power density function.

Since logarithms are to be employed in a resultant graphical display, the function under analysis can be examined in terms of logarithms.

$$(15) \log g^2/\text{Hz} = 2 \log g - \log \text{Hz}$$

If g_{rms} is determined and its logarithm taken, doubling this logarithmic value and subtracting the log of the filter bandwidth will present an averaged value of power spectral density in a logarithmic form. Note that if an average detector with a dynamic range of 40 dB is used (as in Spectral Dynamics Model SD101A Dynamic Analyzer), the action of taking log values and doubling the result gives a true indication of power over four full decades of power. If constant bandwidth filters are employed, the log of filter bandwidth represents a known constant which can be readily subtracted during calibration of an X-Y recorder.

In the analysis of structural response it is seen that in many cases the resonant bandwidths excited at different points on a structure remain essentially constant on a logarithmic scale or in other words are of a constant Q variety. Since it is normally sufficient to resolve a resonant peak in a spectrum with a filter bandwidth which is approximately 1/4 the

width of the resonant peak, it is conceivably useless or redundant to provide excessive resolution at higher frequencies. For this reason, it has long been felt that the ideal spectrum analyzer for use in many investigations of structural properties would be an analyzer whose selectivity or effective filter bandwidth automatically increased with ascending frequency forming a constant Q or constant percentage analysis. If an analyzer were available whose resolution remained essentially constant on a logarithmic scale (a constant Q analyzer) a method of data averaging would have to be selected for use with the filtering function. Two logical choices would exist. In one case, a constant averaging time could be used while on the other hand an averaging time which decreased with increasing filter bandwidth would be most appropriate. This can be more readily seen if we examine some of the statistical considerations involved in the selection of averaging time and analysis rate.

DEGREES OF FREEDOM

In analysis of a periodic waveform, harmonic amplitudes can be considered to exist at some fixed level for an infinite time length. Thus, the averaging of harmonic amplitudes does not become a consideration since the average of a periodic function is the same whether it is averaged over one or many integer periods. However, when performing an analysis of random waveforms, it must be kept in mind that since the data to be processed is essentially random in nature no exact answer to the frequency properties of the signal may really be determined. Rather an estimate of the spectral content of a signal must be achieved. Since the properties of a random signal are governed by statistical considerations the quality of a power spectral density estimate is likewise governed by many statistical properties. One of the primary and most often misused considerations is that of statistical degrees of freedom (n) associated with a data sample.

In one sense the degrees of freedom associated with a section of data are intimately related to the number of samples required to reconstruct the data signal under analysis. Thus, if a d-c signal were to be analyzed, one sample would be sufficient to predict the behavior of the signal for all eternity. By intuition we could say that this data would have very little "freedom". One of the first to extend this reasoning to arbitrary data signals was H. Nyquist of Bell Telephone Laboratories. He showed that the number of samples required to completely define an arbitrary signal was directly related to the bandwidth of the signal. For a data bandwidth B Hz wide he was able to show that 2 B samples were sufficient to allow the signal to be reconstructed in a time domain sense. Thus, if a signal B Hz wide in frequency and existing for T seconds were to be sampled and reconstructed without ambiguity, a total number of samples equal to 2 BT would be required.

This same reasoning follows over directly into the statistical estimation of the properties of random signals. Here we define statistical degrees of freedom

$$(16) \quad n = 2BT$$

where B is the bandwidth of the data signal in Hz and T is the length of the data sample in seconds. It is intuitively obvious that if we restrict the bandwidth of the signal under analysis or reduce the total time over which we may observe the signal, we are in one sense reducing the freedom of the signal. As the freedom of a random signal is reduced, similarly our ability to accurately specify a property of the random signal is also reduced.

As was mentioned previously, it has been shown in many references that the output of a narrow band-pass filter, when random noise is applied to the input, does truly approach a Gaussian distribution. Since power spectral density measurements will be made employing narrow band filtering, we may then say that the immediate output of our analyzing filter will follow a Gaussian density function. Since we are interested in the square of the signal we are then concerned with the properties of the square of a Gaussian random variable. If a random variable exists which follows a Gaussian density function, it has been shown that the square of this variable follows what is known as a Chi-square (χ^2) distribution. As the freedom, or statistical degrees of freedom, associated with our data samples varies, our ability to accurately specify a function of the squared value of the signal we are analyzing is directly related to the Chi-square distribution. This is shown more fully in Figure 3. Note that for a given statistical confidence, our ability to accurately specify the true power spectral density of a signal increases in direct proportion with the statistical degrees of freedom of our signal.

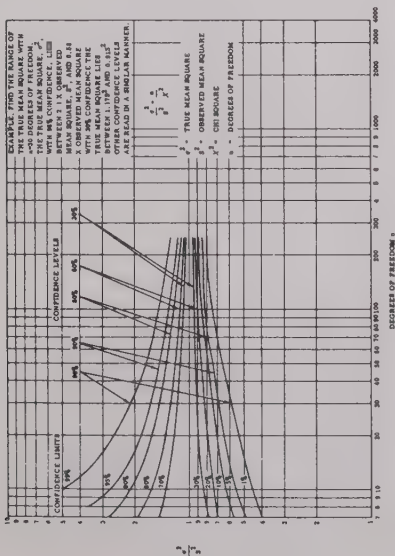


FIGURE 3.

CHI-SQUARE DISTRIBUTION CHART

AVERAGING

Knowing that the statistical freedom of our resultant analysis is based on filter bandwidth and data length, two of the first considerations which must be answered is what length of averaging shall be used and how fast may an analysis be performed. Essentially two types of averaging are available for use in performing PSD estimates. These are true integration over a fixed integration time and continuous RC smoothing or averaging. By using true integration, a point on the PSD estimate is achieved following each integration time. On the other hand, RC averaging uses the weighted results of all signal amplitudes that have gone before. Thus, true integration will result in one plot point after each integration time, whereas continuous or RC averaging will produce a continuous spectral presentation which in many instances is easier to interpret.

It has been shown in Reference 2 (Section 6) that in order to produce an unbiased estimate of a mean squared value, statistically equivalent variances will be achieved if an effective integration time, T, equal to twice the RC circuit time constant is used. This result was shown to be independent of bandwidth B and thus gives an equivalent expression for statistical degrees of freedom as

$$(17) \quad n = 2BT = 4BRC$$

Thus, a time constant $RC = T/2$ can be used. A larger time constant will not improve the statistical quality of the measurement whereas a shorter time constant will definitely reduce the statistical quality of the measurement.

COMPARISON OF ANALYZERS

A. Fixed Bandwidth, Fixed Averaging Time

If a data sample T seconds long were to be analyzed using a bandwidth B cycles wide, intuition alone would indicate that it would be desirable for the entire data sample to pass through the filter bandwidth prior to averaging. Thus, using true integration we would arrive at an analysis or sweep rate of one filter bandwidth per data sample. That is,

$$(18) \quad AR = B/T$$

If RC averaging is used it must be kept in mind that it takes approximately 4 time constants (4 RC seconds) for the output of an RC averaging circuit to build up to 98% of its final output. The maximum analysis rate (sweep rate) for the RC averaging case then becomes

$$(19) \quad AR \leq B/4RC$$

Note that since we have selected $RC = T/2$, this result states that when using true integration a sweep rate twice as fast as that used during RC integration is possible. These sweep rates, of course, apply to the use of a

constant bandwidth filter with resultant linear sweep rate. The total analysis time (A_t) in this case would then be

$$(20) \quad A_t = \frac{1}{AR} [f_2 - f_1]$$

where AR is analysis rate. Using a fixed bandwidth and constant RC averaging time the statistical degrees of freedom would then remain constant over the entire range of analysis.

B. Constant Percentage Bandwidth, Fixed Averaging Time

The equivalent results obtained when using a constant percentage bandwidth analyzer with fixed averaging time have been derived in Reference 3. If p is the percentage of bandwidth with respect to frequency of operation (f_0), the corresponding equations become

$$(21) \quad n = 4pf_0 RC$$

$$(22) \quad AR \leq pf_0/T; \quad AR \leq pf_0/4RC$$

$$(23) \quad A_t = \frac{T}{p} \ln \left[\frac{f_2}{f_1} \right] = \frac{4RC}{p} \ln \left[\frac{f_2}{f_1} \right]$$

This shows that the statistical confidence level increases directly with the center frequency of the analyzer. It also points out that the sweep rate is a first order function of the analysis frequency or is in fact a logarithmic sweep. Thus, we have set up a condition where filter bandwidth and analysis rate are continuously increasing but at the same time statistical confidence is also increasing. If we wish to provide a constant percentage analysis with constant statistical accuracy, a solution would be to provide decreasing RC averaging time as frequency is increased.

C. Constant Percentage Bandwidth, Tracking Averaging Time

This situation has also been examined in Reference 3 and results in a constant percentage type analysis in which bandwidth is continuously increased, sweep rate is continuously increased and averaging time constant is continuously decreased. Although, at the present time, this type of analyzer operating over wide frequency and dynamic range is beyond the state of the art, it is interesting to note the results which are obtained from analysis of such an operation. If k is defined as a constant depending upon the desired statistical confidence level, f_0 is the analyzer filter center frequency and p is the percent bandwidth with respect to f_0 , then the corresponding equations for statistical degrees of

freedom, analysis rate and total analysis time become

$$(24) \quad n = 4kp$$

$$(25) \quad AR = pf_0^2/4k$$

and

$$(26) \quad A_t = \frac{4k}{p} \left[\frac{1}{f_1} - \frac{1}{f_2} \right]$$

where

$$(27) \quad k = f_0 RC$$

D. Programmed Fixed Bandwidth/Variable Averaging Time

There are naturally cases where a constant percentage resolution analyzer is not desirable. This would be the case for example when performing an analysis of periodic waveforms. If a continuously increasing bandwidth filter were used, higher harmonics would tend to become intermingled within the filter bandwidth thus giving erroneous results. A completely versatile analysis system would then of necessity contain both fixed bandwidth filters and programmably increasing bandwidth filters.

From a practical standpoint, such a system has been developed and is the Spectral Dynamics Corporation Model SD1001-2 Automatic Power Spectral Density Analysis System (Figure 4). This system can automatically switch between five preselected constant bandwidth analysis filters. Sweep rate, averaging time constant and bandwidth normalizing division are also automatically programmed, in addition to performing bandswitching, in the SD1001-2 system.

The analysis equations for each of the programmed filters within the SD1001-2 system are the same as for the fixed bandwidth, fixed averaging time situation case A. Thus,

$$(28) \quad RC = \frac{n}{4B}$$

$$(19) \quad AR \leq \frac{B}{4RC}$$

$$(20) \quad A_t = \frac{1}{AR} [f_2 - f_1]$$

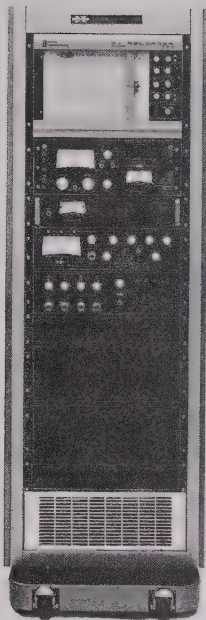


FIGURE 4
SD1001-2

ANALYSIS COMPARISON

As an example, let us examine a typical analysis program as performed on the four types of analysis systems discussed. It is assumed that a random acceleration data sample has been recorded on a tape loop of length T_r equal to 10 seconds. We wish to analyze this from 10 Hz to 2,000 Hz with a desired number of degrees of freedom of 100. A maximum structural Q of 5 is expected which in turn will serve to place some limits on the resolution required in the constant Q type analyzers. Since an analysis bandwidth equal to one-fourth the bandwidth of the narrowest resonance shown in the spectrum should be used, an analysis Q , (Q_A), of 20 is required. This requires a percent resolution of 5%. At the low frequency end of the analysis (10 Hz) this would require an analysis bandwidth of 0.5 Hz.

A. In the first situation, an analysis will be performed using one constant bandwidth filter and no band switching. If a 5 Hz filter is used, minimum resolution is not attained until 100 Hz.

$$(28) \quad Q_A = 20 = \frac{f_0}{B}; \quad f_0 = 20B = 100 \text{ Hz}$$

Naturally above 100 Hz, resolution far better than that required will be the result. With the 5 Hz filter we then calculate an averaging time of 5 seconds ($RC = \frac{n}{4B}$) and a sweep rate of 1/4 Hz per second ($AR = \frac{B}{4RC}$). This results in a total analysis time of 7960 seconds ($A_t = \frac{1}{1/4} [2000 - 10]$) or almost 133 minutes.

This is an extremely impractical result but unfortunately the only type of analysis provided on many older type analyzers.

B. Let us next examine the same analysis if it is performed using a constant percentage analyzer with fixed RC averaging. The statistical degrees of freedom, n , are once again

$$(29) \quad n = 4pf_0RC$$

Since a 5% analysis bandwidth has been specified, let us assume that the minimum desired statistical degrees of freedom $n = 100$ is first achieved at 30 Hz. The necessary RC time constant then becomes

$$(30) \quad RC = \frac{100}{4(0.05)(30)} \approx 16 \text{ seconds}$$

The total analysis time

$$(31) \quad A_t = \frac{4RC}{p} \ln \left[\frac{f_2}{f_1} \right] = \frac{4(16)}{0.05} \ln 200$$

is then 6784 seconds or 113 minutes. This is a slight improvement over the fixed bandwidth situation, although in this case an optimum analysis resolution has been achieved throughout almost the entire display.

C. In the next case, let us examine the constant percentage bandwidth analyzer using tracking RC averaging. As was previously seen, in this case

$$(32) \quad RC = k/f_0$$

where k is a constant depending upon the degree of statistical confidence desired. Then the total degrees of freedom are

$$(33) \quad n = 4pk$$

Again selecting 100 degrees of freedom and a 5% analysis bandwidth $k = \frac{100}{4(0.05)} = 500$. Our

largest averaging time would then again be selected to occur at 30 cycles and would be $RC = k/f_0 = \frac{500}{30}$ or approximately 16 seconds.

Note that at the upper end of the analysis, RC would have reduced to $RC = \frac{R}{f_u} = \frac{500}{2000} = 0.25$ seconds.

The total analysis time in this case is then $A_t = \frac{4k}{p} \left[\frac{1}{f_1} - \frac{1}{f_2} \right]$. This becomes al-

most exactly 4,000 seconds or 66-2/3 minutes. This is approximately half the time required if a single constant bandwidth filter were used for analysis or if a constant percentage analyzer with fixed RC averaging were used.

D. As the final case, we will examine the condition of preprogramming and switching between five fixed bandwidth analysis filters. Assume that available bandwidths are 1.5, 5, 10, 20 and 50

Hz. Although we will not be able to meet all the resolution requirements of a 5% resolution analysis, a realistic compromise can be readily achieved.

The crossover frequencies at which switching between filters should occur is given by

$$(34) \quad f_0 = \frac{B}{P}$$

To meet the minimum resolution requirements using the five available filters, switching should take place at 100, 200, 400 and 1000 Hz. However, in the interest of analysis time and statistical confidence switching will actually be performed at 40, 200, 400 and 1000 Hz. Averaging times are then computed using $RC = \frac{n}{4B}$. We wish to set $n = 100$; however,

for the first filter the maximum "freedom" available is

$$(35) \quad n_{\max} = 2BT_r$$

where T_r is the tape loop length.

The entire set of analysis parameters used to control the SD1001-2 system for this test are shown in the following table:

SD1001-2 PROGRAMMED ANALYSIS PARAMETERS

BW	Freq. Range	n	RC	AR	A _t	Analyzer Calib.
1.5	10 Hz - 40 Hz	30	5	0.075	400	0.7
5	40 Hz - 200 Hz	100	5	0.25	640	.384
10	200 Hz - 400 Hz	100	2.5	1	200	.271
20	400 Hz - 1000 Hz	100	1.25	4	150	.192
50	1000 Hz - 2000 Hz	100	0.5	20	50	.121

Total analysis time = 1440 seconds = 24 minutes

The parameters actually set into the SD1001-2 are then BW, RC, AR, Analyzer Cal and Bandswitching frequencies. The statistical accuracy has been kept essentially constant throughout with a programmed linear sweep rate used for each filter. In this example, we first meet the requirement for maximum resolution at 30 Hz; resolution has been somewhat sacrificed up to that point.

A. Single fixed bandwidth filters - 133 minutes

B. Constant Q fixed RC - 113 minutes

C. Constant Q tracking RC - 67 minutes

D. SD1001-2 System - 24 minutes

COINCIDENT AND QUADRATURE SPECTRUM ANALYSIS

Thus far, the main point of discussion has been that of analyzing one signal for its spectrum, time correlation or amplitude content. It was shown in equation 5 that a power spectrum measurement involves the product of a Fourier transform of a signal and its conjugate transform. Thus, any complex or imaginary component of a signal (with respect to the Fourier weighting function) is offset by an equal but opposite component in the conjugate. The result is that a power spectrum estimate is a real, even function. That is, it is independent of phase. Thus, two signals differing only in phase will have identical power spectra.

However, in many practical cases in engineering, information relating two signals is required. Thus, any time the transfer function between two points in a system is required, the measurement of input/output relationships or between-signal properties is required.

In the frequency domain, one measurement of mutual properties between signals is given by the cross-spectrum or cross-spectral density function. In general, most practical circuits and systems (linear or non-linear) are not made up of purely dissipative components and thus have a complex transfer function. The cross-spectrum properties will then likewise, in most cases, contain complex terms or in other words have both real and imaginary components.

From the definition of cross correlation given by equation 2, one definition of cross-spectral density may be stated as

$$(36) \quad P_{12}(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{12}(\tau) e^{-jw\tau} d\tau$$

The cross-spectral density function $P_{12}(w)$ is thus the Fourier transform of the cross-correlation function $[\phi_{12}(\tau)]$. Since $P_{12}(w)$ is in general complex, it may also be written either

$$(37) \quad P_{12}(w) = C_{12}(w) - jQ_{12}(w)$$

where $C_{12}(w)$ is the co component and $Q_{12}(w)$ is the quad component or as

$$(38) \quad P_{12}(w) = |P_{12}(w)| \angle \phi(w)$$

where

$$(39a) \quad |P_{12}(w)| = (C_{12}(w)^2 + Q_{12}(w)^2)^{1/2}$$

and

$$(39b) \quad \phi(\omega) = \tan^{-1} - \frac{Q_{12}(\omega)}{C_{12}(\omega)}$$

Equation 37 gives the cross spectral density function as a CO and QUAD component whereas in equation 38 the cross spectral density is shown as a magnitude and phase component. These are equivalent expressions.

In practice a presentation in terms of CO and QUAD components is often preferred. This is many times the case because of difficulty in detecting and presenting a phase angle near 0° or 360° (as required in equation 39) and because CO and QUAD components may be plotted versus one another (as shown in Figure 5) to form a Nyquist diagram. Nyquist diagrams are extremely useful in predicting the stability behavior of a system.

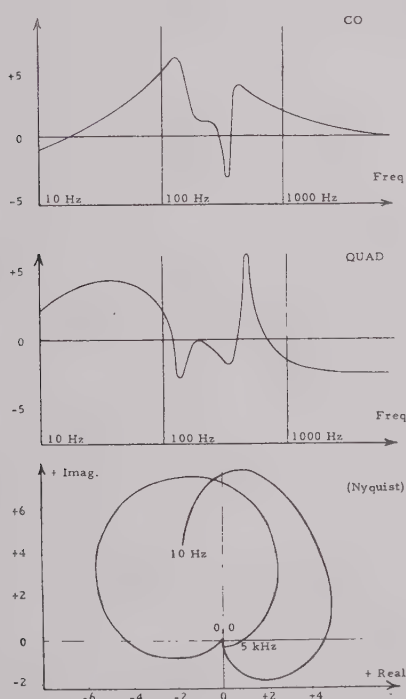


FIGURE 5: CO/QUAD COMPONENT PRESENTATION

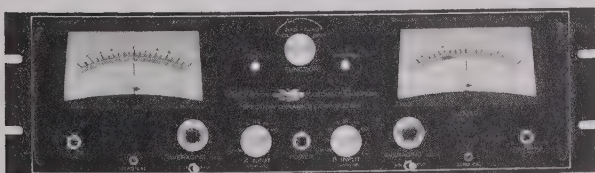


FIGURE 6

SD109

A commercially available analyzer for determining cross spectrum information between 2 signals is the Model SD109 CO/QUAD Analyzer available from Spectral Dynamics Corporation. Designed to operate with two Spectral Dynamics Tracking Filters, the Model SD109 operates on both random and periodic signals and presents two outputs in terms of in phase and quadrature components between signals. The SD109 is shown in Figure 6.

CONCLUSIONS

We have thus seen that although three types of analysis (spectrum, correlation and probability) are quite useful, several reasons dictate the more broad usefulness of spectrum analysis over correlation and probability. Having decided on the performance of a spectrum analysis, it has been shown that in general the most useful term for describing the spectral content of an arbitrary random signal is power spectral density in terms of g_{rms}^2/Hz . Several types of analyzers have then been investigated including constant bandwidth, fixed RC averaging; constant percentage bandwidth with fixed RC averaging; constant percentage bandwidth with tracking RC averaging; and multi-filter fixed bandwidth analyzers with programmed averaging time constant.

The use of constant percentage bandwidth analyzers with tracking RC averaging in one sense represents the ideal analyzer in terms of optimum resolution constant statistical accuracy and minimum analysis time. On the other hand, the use of a programmable fixed bandwidth analyzer using linear detectors in a logarithmic display of g^2/Hz was seen to provide excellent dynamic range and a remarkable saving in total time required for a specific analysis.

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APPENDIX

Relationships Between Average + RMS Values

A. SINEWAVE

For a sinewave of peak amplitude A defined by

$$f(t) = A \sin \omega t$$

an average value, f_{ave} , may be derived as

$$f_{ave} = \frac{1}{T} \int_0^T A \sin \omega t \, dt$$

where T is the period of the sinewave.

Then

$$\begin{aligned} f_{ave} &= \frac{2}{T} \int_0^{T/2} A \sin \omega t \, dt \\ &= \frac{2}{T} \left[-\frac{A \cos \omega t}{\omega} \right]_0^{T/2} \end{aligned}$$

since $\omega = 2\pi f = \frac{2\pi}{T}$ this becomes

$$\begin{aligned} f_{ave} &= \frac{2AT}{T2\pi} \left[-\cos \frac{2\pi}{T} \cdot \frac{T}{2} + \cos 0 \right] \\ &= \frac{A}{\pi} [2] = \frac{2A}{\pi} = 0.636A \end{aligned}$$

Similarly a root mean square value f_{rms} may be derived as

$$\begin{aligned} f_{rms} &= \left(\frac{1}{T} \int_0^T [A \sin \omega t]^2 dt \right)^{1/2} \\ &= \left(\frac{1}{T} \int_0^T [A^2 \sin^2 \omega t] dt \right)^{1/2} \\ &= \left(\frac{A^2}{T} \int_0^T \frac{[1 - \cos 2\omega t]}{2} dt \right)^{1/2} \\ &= \left(\frac{A^2}{2T} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \right)^{1/2} \\ &= \left(\frac{A^2}{2T} \left[T - \frac{\sin 2\pi T}{\frac{4\pi}{T}} + \sin 0 \right] \right)^{1/2} \\ &= \left(\frac{A^2}{2} \right)^{1/2} = \frac{A}{\sqrt{2}} = 0.707A \end{aligned}$$

Thus the ratio of rms to average value for a sinewave is

$$\frac{0.707A}{0.636A} = 1.11$$

Note that the mean square value is $A^2/2$ and the square of the mean (average) value is $\frac{4A^2}{\pi^2}$.

mean square = $0.5A^2$
square of mean = $0.4053 A^2$

$$\begin{aligned} \text{ratio of mean square to square of mean} &= \\ 1.23 &= (1.11)^2 \end{aligned}$$

B. GAUSSIAN RANDOM VARIABLE

The Gaussian density function is described by

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where x is the instantaneous amplitude of the random variable under analysis, μ is its mean value and σ^2 will be shown to be the variance from the mean.

That the mean value of the Gaussian distribution is truly μ can be shown by solving for the first moment, m_1 . Thus,

$$\begin{aligned} m_1 &= \int_{-\infty}^{+\infty} xp(x) \, dx \\ &= \int_{-\infty}^{+\infty} \frac{x e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \, dx \end{aligned}$$

To solve this integral, a change of variables is required such that

$$y = \frac{(x-\mu)}{\sigma\sqrt{2}} ; \, dx = \sqrt{2}\sigma dy$$

Then

$$m_1 = \int_{-\infty}^{+\infty} \frac{(\sqrt{2}\sigma y + \mu)}{\sqrt{\pi}} e^{-y^2} dy$$

since

$$\int_{-\infty}^{+\infty} y e^{-y^2} dy \equiv 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} e^{-y^2} dy \equiv \sqrt{\pi}$$

$$m_1 = \frac{\mu}{\sqrt{\pi}} \cdot \sqrt{\pi} = \mu \quad \text{and the mean value is}$$

truly μ .

The second central moment or variance, μ_2 , is

$$\mu_2 = \int_{-\infty}^{+\infty} \frac{(x-\mu)^2 e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \, dx$$

using the same change of variables as above

$$\begin{aligned} \mu_2 &= 2 \int_{-\infty}^{+\infty} \frac{\sqrt{2}\sigma y^2}{\sigma\sqrt{2\pi}} e^{-y^2} dy \\ &= \frac{4\sigma^2}{\sqrt{\pi}} \int_0^{\infty} y^2 e^{-y^2} dy \end{aligned}$$

As shown in Reference 5, page 68

$$\int_0^{\infty} y^2 e^{-y^2} dy \equiv \frac{\sqrt{\pi}}{4}$$

thus

$$\mu_2 = \frac{4\sigma^2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{4} = \sigma^2$$

The standard deviation (rms value) is then

$$x_{\text{rms}} = (\sigma^2 - \mu^2)^{1/2}$$

For a mean value of zero, the rms value is then simply σ .

From the symmetry of the distribution, the rectified average value can be found by

$$x_{\text{ave}} = \frac{2}{\sqrt{2\pi}} \int_0^{\infty} \frac{x}{\sigma} e^{-x^2/2\sigma^2} dx$$

assuming a mean value, $\mu = 0$

substituting $y = x/\sigma\sqrt{2}$

$$\begin{aligned} x_{\text{ave}} &= 2 \sqrt{\frac{2}{\pi}} \sigma \int_0^{\infty} y e^{-y^2} dy \\ &= 2 \sigma \sqrt{\frac{2}{\pi}} \left[\frac{e^{-y^2}}{(-2)} \right]_0^{\infty} \\ &= \sqrt{\frac{2}{\pi}} \sigma = 0.798 \sigma = x_{\text{ave}} \end{aligned}$$

Thus, the ratio of rms to average value for a random signal following a Gaussian density function is

$$\frac{\text{rms}}{\text{avg}} = \frac{\sigma}{0.798 \sigma} = 1.25$$

The ratio of the mean square to the square of the (rectified or absolute) mean is then

$$\sigma^2 = (1.25 x_{\text{ave}})^2$$

$$\sigma^2 = 1.57 x_{\text{ave}}^2$$

